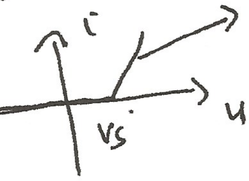
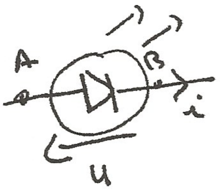
Correction DS n°2 14/10/22.

1.

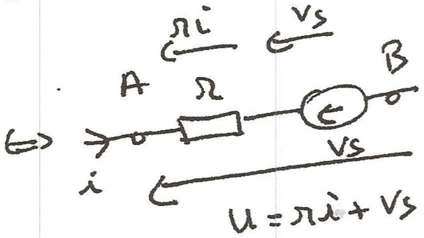


$\rightarrow$ droite $i = a_1 u + b_1$ ou $u = a i + b$.

avec $a = r$ et $b = u(i=0) = V_s$.



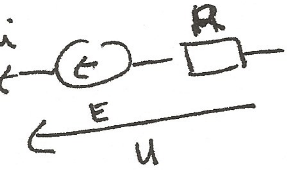
$u = ri + V_s \Rightarrow$



$u = ri + V_s$

Pour $u < V_s$ $i = 0$ $\Rightarrow$ —▷|— $\Leftrightarrow$ —/ —.

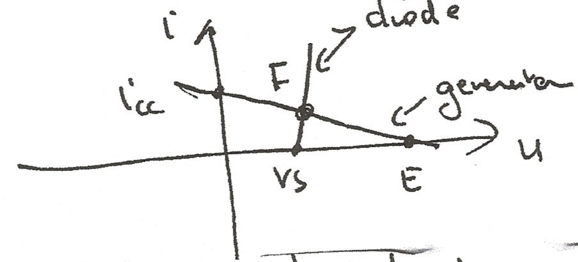
Point de fonctionnement.



$u = E - Ri \rightarrow i = \frac{E}{R} - \frac{u}{R}$. $i_{cc} = \frac{E}{R}$

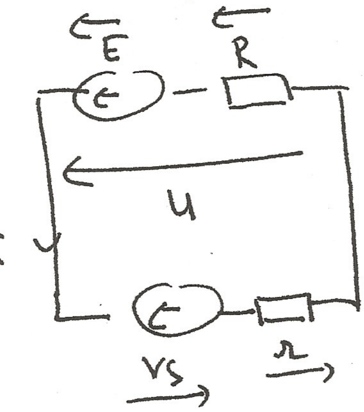
$u_0 = E = 6V > u_s$

Trace les 2 caractéristiques sur le m graphe



Le point de fonctionnement F. se situe sur la partie passante de la diode.

On remplace la diode par son modèle de Thevenin équivalent.



loi des maille.

$E - (R+r)i - V_s = 0$

$$\boxed{i = \frac{E - V_s}{R + r}.}$$

Donc $\boxed{R = \frac{E - V_s}{i} - r.}$

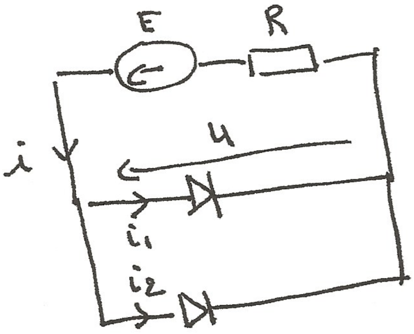
AN $i_0 = 15\,mA = 15 \cdot 10^{-3} A$.

$E = 6V$ $r = 1\,\Omega$.

$V_s = 1{,}8V$

$\frac{6 - 1{,}8}{15 \cdot 10^{-3}} - 1 = \boxed{280\,\Omega = R_0.}$

c]



Diodes identiques $i_1 = i_2 = i_0$

et $\begin{cases} i = 2i_0 . \\ u = +V_S + r i_0 . \end{cases}$

Loi des mailles : $E - R(2i_0) = V_S + r i_0 \Rightarrow R = \frac{E - V_S - r i_0}{2 i_0}.$

$$\boxed{R'_0 = \frac{1}{2}\left[\frac{E - V_S}{i_0} - r\right] = \frac{R_0}{2}}$$

AN $R'_0 = 140\,\Omega$.

d] $P_R = R'_0 \cdot (2i_0)^2 = \boxed{4 R'_0 i_0^2 = P_R}$ AN $\boxed{P_R = 0{,}13\,W}$

$P_d = (V_S + r i_0)\, i_0 = \boxed{V_S \cdot i_0 + r \cdot i_0^2 = P_d}$ $\boxed{P_d = 0{,}027\,W}$

$P_G = P_R + 2\cdot P_d$ car $\boxed{P_G = 0{,}18\,W}$ u. $E - R\cdot 2i_0 - u = 0$

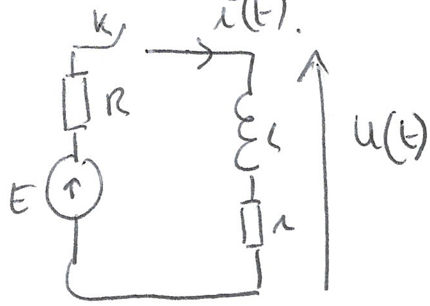
$[E = 2Ri_0 + u] \times 2i_0$

$$\underbrace{E\cdot 2i_0}_{P_G} = \underbrace{R\cdot(2i_0)^2}_{P_R} + \underbrace{u\cdot 2i_0}_{2P_d}$$

Rq $P_G > 0$ en convention récepteur.

Ex 2

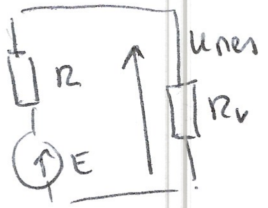
Partie 1.



Question préliminaire : Méthode de la "tension moitié"

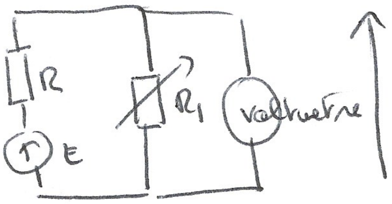
Générateur réel.

1ère mesure → On mesure E = tension à vide à l'aide d'un voltmètre de résistance interne $R_v \gg R$.



$$U_{mes} = \frac{R_v}{\cancel{R}+R_v} \cdot E \simeq E.$$

2e mesure.



$$U_{mes} = \frac{R_1}{R+R_1} \cdot E.$$

Lorsque $R_1 = R \Rightarrow U_{mes} = E$.

On relève la valeur de R_1 pour en déduire celle de E.

1] K fermé. $E = (R+r)i + L\frac{di}{dt} \rightarrow \frac{di}{dt} + \frac{R+r}{L} i = \frac{E}{L}$.

2) $\boxed{\tau = \frac{L}{R+r}}$ $\quad i(10\tau) \rightarrow i(\infty)$ régime permanent. et $i(\infty) = S_p = \frac{E}{R+r}$

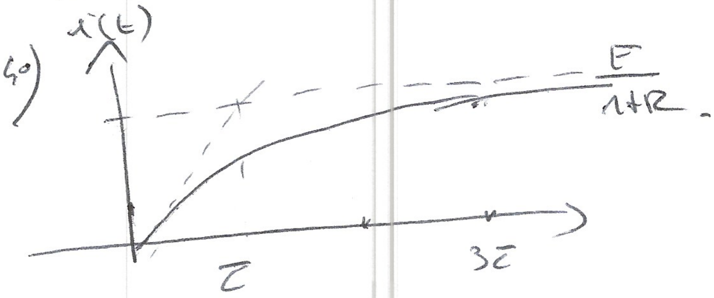
AN $\tau = \frac{0{,}1}{60} = 16 \cdot ms$

3] $i(t) = Ae^{-\frac{t}{\tau}} + \frac{E}{R+r}$ $\quad$ Continuité de i ds la bobine. a $i(0^-) = 0 \stackrel{\downarrow}{=} i(0^+) = 0$.

K ouvert

$$\rightarrow \boxed{i(t) = \frac{E}{R+r}\left(1 - \exp^{-\frac{t}{\tau}}\right)}$$

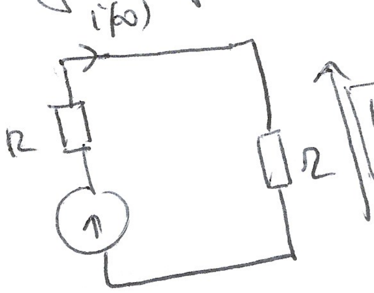
4°)



5°] La loi des mailles

à $t=0^+$: $E = Ri + u$ à l'instant $t=0^+$ donne $\boxed{u(0^+) = E}$.

en régime permanent —m = ——



$$\boxed{u(\infty) = \frac{r}{R+r} \cdot E}$$

b] $u = ri + L\frac{di}{dt} = \frac{r \cdot E}{R+r}\left(1 - \exp^{-\frac{t}{\tau}}\right) + L \cdot \frac{E}{R+r} \cdot \frac{1}{\tau} \exp^{-\frac{t}{\tau}}$

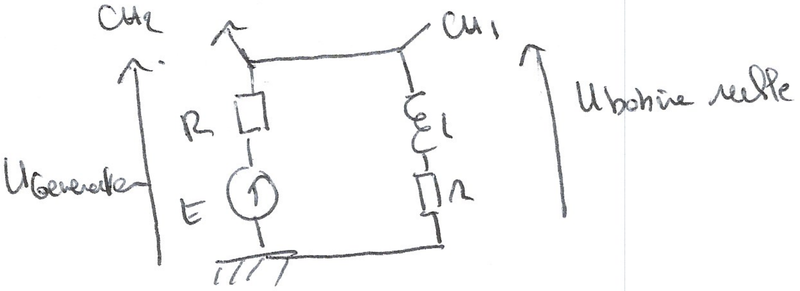
$$= \frac{r}{r+R} \cdot E - \frac{r E e^{-\frac{t}{\tau}}}{R+r} + E e^{-\frac{t}{\tau}} = \frac{r}{R+r} \cdot E + E\left(1 - \frac{r}{R+r}\right) e^{-\frac{t}{\tau}}$$

$$\boxed{u(t) = \frac{r}{R+r} \cdot E + \frac{R}{R+r} \cdot E e^{-\frac{t}{\tau}}}$$

à $t=0$, on obtient $u(0^+) = \frac{r+R}{r+R} \cdot E = E$

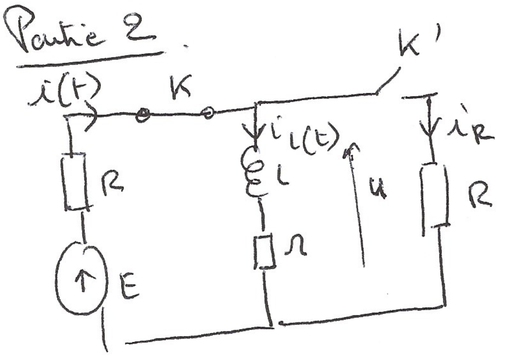
$t \to \infty$ $\quad u(\infty) = \frac{r}{R+r} \cdot E$

résultat du 5a sont bien vérifiés



⇒ On observe donc les 2 courbes sur CH1 et CH2.
(1 seule suffit donc).

Partie 2

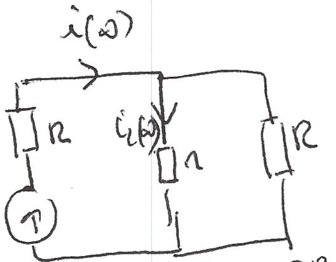


1- à $t=0$, on ferme K'

$t=0^-$, régime permanent établi ⇒

$i_L(0^-) = \frac{E}{R+r}$ et par continuité de i ds L ⇒ $\boxed{i_L(0^+) = \frac{E}{R+r}}$

en régime permanent



$$i(\infty) = \frac{E}{R+\frac{rR}{r+R}} = \frac{r+R\cdot E}{R^2+2rR}$$

Diviseur de courant $\frac{i_L(\infty)}{i(\infty)} = \frac{\frac{rR}{r+R}}{r} = \frac{R}{r+R}$

$$\Rightarrow \boxed{i_L(\infty) = \frac{\cancel{R}\cdot E}{\cancel{R}(R+2r)}}$$

2- équation diff à t sur i_L

$$\begin{cases} E = Ri + u \quad (1) \\ i = i_L + i_R \quad (2) \\ u = L\frac{di_L}{dt} + r i_L = R i_R \quad (3) \end{cases}$$

(2) +(1) +3 → $\frac{E-u}{R} = i_L + \frac{u}{R}$ → $\frac{E}{R} = i_L + \frac{2u}{L}$ ⇒ $\frac{E}{R} = i_L + \frac{2}{L}\left(L\frac{di_L}{dt} + r i_L\right)$

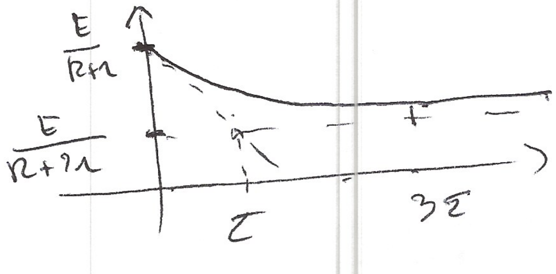
$$\Rightarrow \frac{di_L}{dt} + \frac{R+2r}{2L} i_L = \frac{E}{2\cancel{L}}$$

$\boxed{\tau = \frac{2L}{R+2r}}$ $S_p = \frac{E}{R+2r} = i_L(\infty)$!

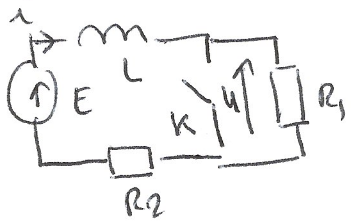
$$i_L(t) = A e^{-\frac{t}{\tau}} + \frac{E}{R+2r}$$

$$i_L(0^+) = \frac{E}{R+r} = A + \frac{E}{R+2r} \rightarrow A = \frac{E}{R+r} - \frac{E}{R+2r} = E\left[\frac{\overbrace{\cancel{R}+2r-\cancel{R}-r}^{r}}{(R+r)(R+2r)}\right]$$

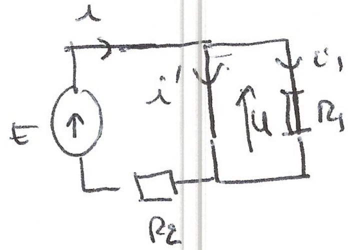
$$\boxed{i_L(t) = \frac{E}{R+2r}\left[1 + \frac{r}{R+r} e^{-\frac{t}{\tau}}\right]}$$



Partie 3



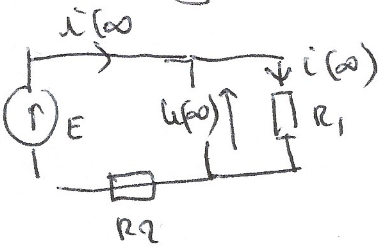
1] K fermé $\Rightarrow$ $u = 0$
depuis longtemps
$\Rightarrow$ —m— $\Leftrightarrow$ —



d'où $u = R_1 i_1 \Rightarrow i_1 = 0$ et $i = i' = \dfrac{E}{R_2}$.

2] $t = 0 \Rightarrow$ K ouvert, on a alors $i(0^-) = \dfrac{E}{R_2} = i(0^+)$ continuité de i ds une bobine.

$t \to \infty$ régime permanent. —m— $\Leftrightarrow$ —



$$\boxed{i(\infty) = \frac{E}{R_1+R_2} = i_1}$$

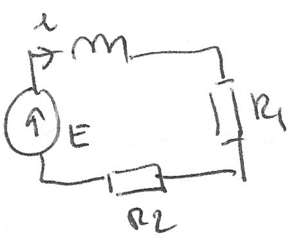
$$\boxed{u(\infty) = \frac{R_1}{R_1+R_2} \cdot E = u_1}$$

3] $\boxed{i(0^+) = \dfrac{E}{R_2}} \Rightarrow \boxed{u(0^+) = \dfrac{R_1}{R_2} \cdot E}$.

AN $\boxed{i(0^+) = 10\,mA}$ $\boxed{u(0^+) = 5.0 \cdot 10^2\,V}$ Valeur élevée.

4] R_1 absente $\Rightarrow$ circuit ouvert. $\Rightarrow R_1 \to \infty$ $u(0^+) \to \infty$.
L'air entre les bornes de l'interrupteur est ionisé et devient conducteur $\Rightarrow$ production d'étincelle.

5]



$$E = L\frac{di}{dt} + (R_1+R_2)\,i$$

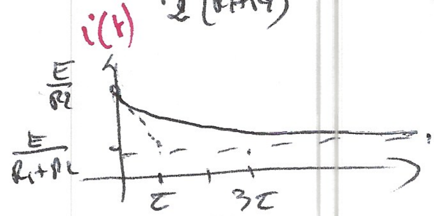
$$\frac{di}{dt} + \frac{R_1+R_2}{L}\, i = \frac{E}{L}.$$

$$\left\{ \begin{array}{l} \tau = \dfrac{L}{R_1+R_2} \\ S_p = \dfrac{E}{R_1+R_2} \end{array} \right.$$

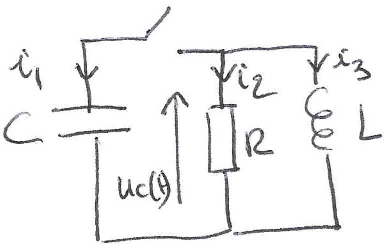
$$i(t) = A e^{-\frac{t}{\tau}} + \frac{E}{R_1+R_2}$$

à $t = 0^+$ $i = \dfrac{E}{R_2} \Rightarrow A = \dfrac{E}{R_2} - \dfrac{E}{R_1+R_2} = \dfrac{R_1 \cdot E}{R_2(R_1+R_2)}$

$$i(t) = \frac{R_1 E}{R_2(R_1+R_2)} e^{-\frac{t}{\tau}} + \frac{E}{R_1+R_2}$$



EX3.



$$\begin{cases} i_1 = C\dfrac{du_c}{dt} & (1) \\ u_c = R i_2 & (2) \\ u_c = L\dfrac{di_3}{dt} & (3) \end{cases}$$

$i_1 + i_2 + i_3 = 0$ (4)

Continuité de u_c aux bornes de C

1°] $u_c(0^-) = \boxed{u_0 = u_c(0^+)}$

$i_3(0^-) = \boxed{0 = i_3(0^+)}$

continuité de i traversant une bobine.

(2) $\Rightarrow$ $\boxed{i_2(0^+) = \dfrac{u_0}{R}}$ et (4) $\Rightarrow$ $\boxed{i_1(0^+) = -\dfrac{u_0}{R}}$

2°] (4) + (1) + (2) $\Rightarrow$ $C\dfrac{du_c}{dt} + \dfrac{u_c}{R} + i_3 = 0$. On dérive + (3)

$C\dfrac{d^2u_c}{dt^2} + \dfrac{1}{R}\dfrac{du_c}{dt} + \dfrac{u_c}{L} = 0$ $\Rightarrow$ $\begin{cases} \dfrac{d^2u_c}{dt^2} + \dfrac{1}{RC}\dfrac{du_c}{dt} + \dfrac{u_c}{LC} = 0 \\ \dfrac{d^2u_c}{dt^2} + 2\lambda\omega_0\dfrac{du_c}{dt} + \omega_0^2 u_c = 0. \end{cases}$

$\omega_0^2 = \dfrac{1}{LC}$ $\Rightarrow$ $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$ $\quad 2\lambda\omega_0 = \dfrac{1}{RC}$ $\Rightarrow$ $\lambda = \dfrac{1}{2RC\omega_0} = \dfrac{\sqrt{LC}}{2RC} = \boxed{\dfrac{1}{2R}\sqrt{\dfrac{L}{C}} = \lambda}$

3°] Régime critique correspond à $\lambda = 1$ $\Rightarrow$ $\dfrac{1}{2R_c}\sqrt{\dfrac{L}{C}} = 1$.

$\boxed{R_c = \dfrac{1}{2}\sqrt{\dfrac{L}{C}}}$ AN $R_c = \dfrac{1}{2}\sqrt{\dfrac{10^{-3}}{4\cdot 10^{-9}}} = \dfrac{10^3}{4} = 0{,}25\cdot 10^3 = \boxed{250\,\Omega = R_c}$

équations aux dimensions $\left[\dfrac{d^2u_c}{dt^2}\right] + 2\lambda\omega_0\left[\dfrac{du_c}{dt}\right] + \omega_0^2[u_c] = 0$

$\dfrac{[\text{Tension}]}{T^2} + 2\lambda\omega_0\dfrac{[\text{Tension}]}{T} + \omega_0^2[\text{Tension}]$ $\Rightarrow [\omega_0^2] = T^{-2}$ $\rightarrow [2\lambda\omega_0] = T^{-1}$

$\Rightarrow [\omega_0] = T^{-1}$

$[\lambda] = 1$

Vérification: $[RC] = T$ $\Rightarrow$ $[C] = \dfrac{T}{[R]}$; $\left[\dfrac{L}{R}\right] = T$ $\Rightarrow$ $[L] = T\cdot[R]$ $\Big\}$ $[LC] = T^2 \Rightarrow [\omega_0] = T^{-1}$ et $\left[\dfrac{L}{C}\right] = [R^2] \Rightarrow [\lambda] = 1$

4]

$$\frac{d^2 u_c}{dt^2} + 2\lambda\omega_0 \frac{du_c}{dt} + \omega_0^2 u_c = 0.$$

e^{rt} solution $\rightarrow$ $r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$$\Delta = 4\omega_0^2(\lambda^2 - 1)$$

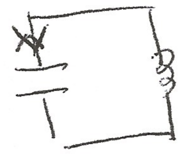
$\lambda < 1 \rightarrow$ Regime pseudo periodique $\Rightarrow u_c(t) = A e^{-\lambda\omega_0 t} \cos(\omega_0\sqrt{1-\lambda^2}\,t + \varphi)$

Les oscillations seront non amorties si $\lambda = 0$ $\Rightarrow \omega = \omega_0$.

et $T = T_0 = \frac{2\pi}{\omega_0}$. $\lambda = 0 \Rightarrow R \to \infty$ (R debranchée).

$u_c(t) = A(\cos(\omega_0 t + \varphi))$

Continuite de i ds L

CI 

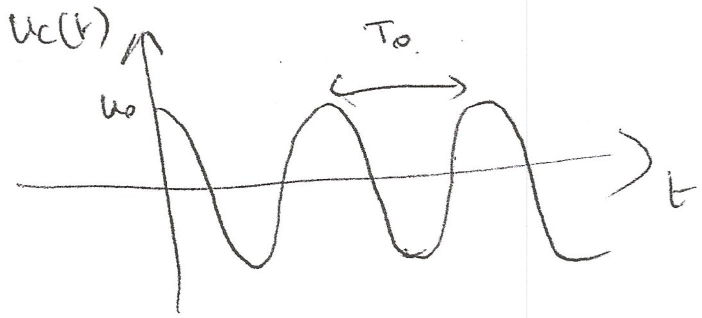
$i = C\frac{du_c}{dt}$ $\quad i(0^+) \overset{\Downarrow}{=} 0 = i(0^-)$ $\Rightarrow \frac{du_c}{dt}(0^+) = 0.$

$\uparrow$ Kouvert

$u_c(0^+) = A\cos\varphi = u_0.$ $\Rightarrow A = u_0$

$\frac{du_c}{dt} = -A\omega_0 \sin\varphi = 0 \rightarrow \varphi = 0$

$$\boxed{u_c(t) = u_0 \cos\omega_0 t.}$$



$R > R_c \Rightarrow \lambda < 1$ Regime pseudo-périodique.

$u_c(t) = A e^{-\lambda\omega_0 t} \cos(\omega_0\sqrt{1-\lambda^2}\, t + \varphi)$ A et φ à determiner avec les

CI $u_c(0^+) = u_0$ et $\frac{du}{dt}(0^+) = \frac{i_1(0^+)}{C} = -\frac{u_0}{RC} = -2\lambda\omega_0 u_0$

or $\frac{du_c}{dt} = A e^{-\lambda\omega_0 t}\left[-\lambda\omega_0 \cos(\omega_0\sqrt{1-\lambda^2}\, t + \varphi) - \omega_0\sqrt{1-\lambda^2} \sin(\omega_0\sqrt{1-\lambda^2}\, t + \varphi)\right]$.

On obtient $u_c(0^+) = A\cos\varphi = u_0$

$$\frac{du_c}{dt}(0^+) = A\left[-\lambda\omega_0\cos\varphi - \omega_0\sqrt{1-\lambda^2}\sin\varphi\right] = -2\lambda\omega_0 u_0.$$

d'où $-\underbrace{A\cos\varphi}_{u_0}\cdot\cancel{\omega_0}\left[\lambda + \sqrt{1-\lambda^2}\,tg\varphi\right] = -2\lambda\cancel{\omega_0}u_0$.

$$\lambda + \sqrt{1-\lambda^2}\,tg\varphi = 2\lambda \Rightarrow tg\varphi = \frac{\lambda}{\sqrt{1-\lambda^2}}$$

$\boxed{\varphi = \arctan\frac{\lambda}{\sqrt{1-\lambda^2}}}$ et $\frac{1}{\cos^2\varphi} = 1 + tg^2\varphi = 1 + \frac{\lambda^2}{1-\lambda^2} = \frac{1}{1-\lambda^2} \rightarrow \cos\varphi = \sqrt{1-\lambda^2}$

$\Rightarrow$ $\boxed{A = \frac{u_0}{\sqrt{1-\lambda^2}}}$ $\boxed{u_c(t) = \frac{u_0 e^{-\lambda\omega_0 t}}{\sqrt{1-\lambda^2}}\cos(\omega_0\sqrt{1-\lambda^2}\, t + \varphi)}$

$\lambda < 1 \Rightarrow A > u_0$