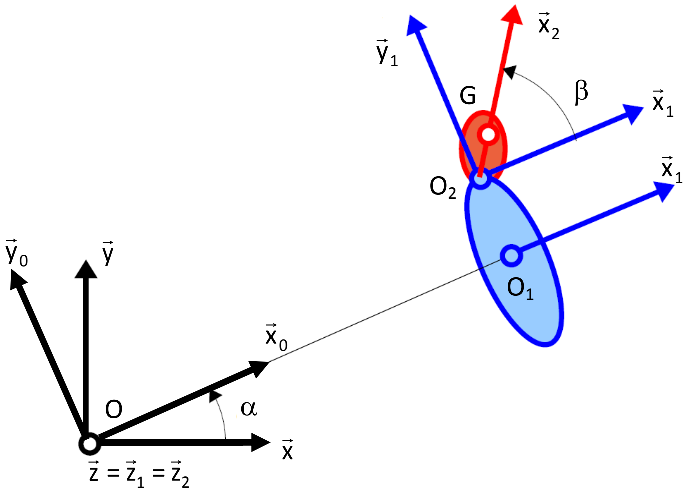
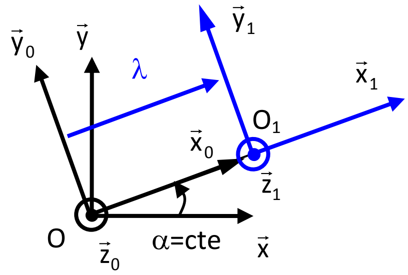
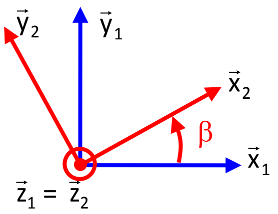
Système de lancement du Space Mountain® - Corrigé

Q.1.





$\overrightarrow{\Omega_{1/0}} = \vec{0}$



$\overrightarrow{\Omega_{2/1}} = \dot{\beta}.\vec{z}_2 \rightarrow \overrightarrow{\Omega_{2/0}} = \dot{\beta}.\vec{z}_2$

Q.2. $\overrightarrow{V_{O_2,2/0}} = \frac{d}{dt}\overrightarrow{OO_2}\Big|_0 = \frac{d}{dt}\left(\lambda.\vec{x}_1 + a_1.\vec{x}_1 + b_1.\vec{y}_1\right)\Big|_0 = \dot{\lambda}.\vec{x}_1 \rightarrow \boxed{\overrightarrow{V_{O_2,2/0}} = \dot{\lambda}.\vec{x}_1}$

Q.3. $\overrightarrow{V_{G,2/0}} = \frac{d}{dt}\overrightarrow{OG}\Big|_0 = \frac{d}{dt}\left(\lambda.\vec{x}_1 + a_1.\vec{x}_1 + b_1.\vec{y}_1 + a_2.\vec{x}_2\right)\Big|_0 = \dot{\lambda}.\vec{x}_1 + a_2.\frac{d}{dt}\vec{x}_2\Big|_0 = \dot{\lambda}.\vec{x}_1 + a_2.\dot{\beta}.\vec{y}_2$

$\rightarrow \boxed{\overrightarrow{V_{G,2/0}} = \dot{\lambda}.\vec{x}_1 + a_2.\dot{\beta}.\vec{y}_2}$

$\overrightarrow{\Gamma_{G,2/0}} = \frac{d}{dt}\overrightarrow{V_{G,2/0}}\Big|_0 = \frac{d}{dt}\left(\dot{\lambda}.\vec{x}_1 + a_2.\dot{\beta}.\vec{y}_2\right)\Big|_0 = \ddot{\lambda}.\vec{x}_1 + a_2.\ddot{\beta}.\vec{y}_2 + a_2.\dot{\beta}.\frac{d}{dt}\vec{y}_2\Big|_0 = \ddot{\lambda}.\vec{x}_1 + a_2.\ddot{\beta}.\vec{y}_2 - a_2.\dot{\beta}^2.\vec{x}_2$

$\rightarrow \boxed{\overrightarrow{\Gamma_{G,2/0}} = \ddot{\lambda}.\vec{x}_1 + a_2.\ddot{\beta}.\vec{y}_2 - a_2.\dot{\beta}^2.\vec{x}_2}$

Rappels :

$\frac{d}{dt}\vec{x}_2\Big|_0 = \frac{d}{dt}\vec{x}_2\Big|_2 + \overrightarrow{\Omega_{2/0}} \wedge \vec{x}_2 = \dot{\beta}.\vec{z}_2 \wedge \vec{x}_2 = \dot{\beta}.\vec{y}_2$

$\frac{d}{dt}\vec{y}_2\Big|_0 = \frac{d}{dt}\vec{y}_2\Big|_2 + \overrightarrow{\Omega_{2/0}} \wedge \vec{y}_2 = \dot{\beta}.\vec{z}_2 \wedge \vec{y}_2 = -\dot{\beta}.\vec{x}_2$

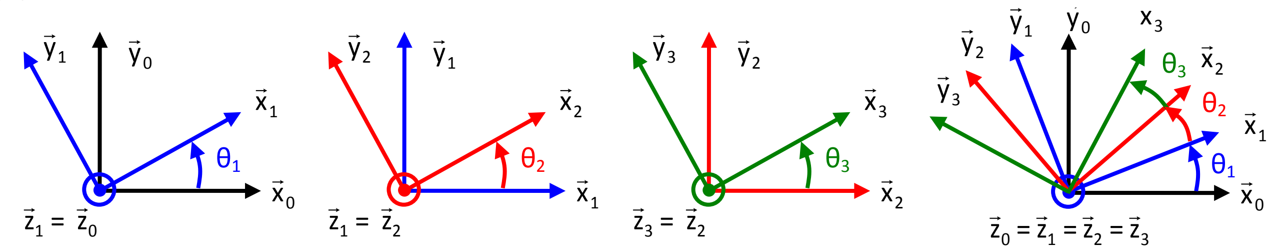
(ici, il est cependant préférable de trouver ce résultat par les figures géométrales)

Q.4. Accélération maximale = 9 m/s² d'après le C.d.C.F. → $a_2.\ddot{\beta} = 9$ → $\ddot{\beta} = \frac{9}{a_2} = \frac{9}{0,17} = 53\ rad.s^{-2}$

$\ddot{\beta} = 53\ rad.s^{-2} < 80\ rad.s^{-2}$ → C.d.C.F. ok

Robot ramasseur de fruits - Corrigé

Q.1.



$\overrightarrow{\Omega_{1/0}} = \dot{\theta}_1.\vec{z}_0$ $\overrightarrow{\Omega_{2/1}} = \dot{\theta}_2.\vec{z}_0$ $\overrightarrow{\Omega_{3/2}} = \dot{\theta}_3.\vec{z}_0$

Q.2. $\overrightarrow{V_{O_1,1/0}} = \overrightarrow{V_{O_1/0}} = \frac{d}{dt}\overrightarrow{O_0O_1}\Big|_0 = \frac{d}{dt}R.\vec{x}_1\Big|_0 = R.\dot{\theta}_1.\vec{y}_1 \rightarrow \boxed{\overrightarrow{V_{O_1,1/0}} = R.\dot{\theta}_1.\vec{y}_1}$

Rappel : $\frac{d}{dt}\vec{x}_1\Big|_0 = \frac{d}{dt}\vec{x}_1\Big|_1 + \overrightarrow{\Omega_{1/0}} \wedge \vec{x}_1 = \left(\dot{\theta}_1.\vec{z}_0\right) \wedge \vec{x}_1 = \dot{\theta}_1.\vec{y}_1$ (ici, il est cependant préférable de trouver ce résultat par les figures 2D de repérage paramétrage)

Q.3. $\overrightarrow{V_{O_2,2/0}} = \overrightarrow{V_{O_2/0}} = \frac{d}{dt}\overrightarrow{O_0O_2}\Big|_0 = \frac{d}{dt}\left(R.\vec{x}_1 + R.\vec{x}_2\right)\Big|_0 = R.\dot{\theta}_1.\vec{y}_1 + R.\left(\dot{\theta}_1 + \dot{\theta}_2\right)\vec{y}_2 \rightarrow \boxed{\overrightarrow{V_{O_2,2/0}} = R.\dot{\theta}_1.\vec{y}_1 + R.\left(\dot{\theta}_1 + \dot{\theta}_2\right)\vec{y}_2}$

Rappel : $\frac{d}{dt}\vec{x}_2\Big|_0 = \frac{d}{dt}\vec{x}_2\Big|_2 + \overrightarrow{\Omega_{2/0}} \wedge \vec{x}_2 = \left(\dot{\theta}_1 + \dot{\theta}_2\right).\vec{z}_0 \wedge \vec{x}_2 = \left(\dot{\theta}_1 + \dot{\theta}_2\right).\vec{y}_2$ (ici, il est cependant préférable de trouver ce résultat par les figures 2D de repérage paramétrage)

Q.4. $\overrightarrow{V_{M,3/0}} = \overrightarrow{V_{M/0}} = \frac{d}{dt}\overrightarrow{O_0M}\Big|_0 = \frac{d}{dt}\left(R.\vec{x}_1 + R.\vec{x}_2 + L.\vec{x}_3\right)\Big|_0 = R.\dot{\theta}_1.\vec{y}_1 + R.\left(\dot{\theta}_1 + \dot{\theta}_2\right)\vec{y}_2 + L.\left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right)\vec{y}_3$

$\rightarrow \boxed{\overrightarrow{V_{M,3/0}} = R.\dot{\theta}_1.\vec{y}_1 + R.\left(\dot{\theta}_1 + \dot{\theta}_2\right)\vec{y}_2 + L.\left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right)\vec{y}_3}$

Rappel : $\frac{d}{dt}\vec{x}_3\Big|_0 = \frac{d}{dt}\vec{x}_3\Big|_3 + \overrightarrow{\Omega_{3/0}} \wedge \vec{x}_3 = \left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right).\vec{z}_0 \wedge \vec{x}_3 = \left(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3\right).\vec{y}_3$ (ici, il est cependant préférable de trouver ce résultat par les figures 2D de repérage paramétrage)

Q.5. On a $\theta_2 = \pi - 2.\theta_1 \rightarrow \dot{\theta}_2 = -2.\dot{\theta}_1$ et $\theta_3 = \theta_1 - \frac{\pi}{2} \rightarrow \dot{\theta}_3 = \dot{\theta}_1$

D'où : $\overrightarrow{V_{M,3/0}}.\vec{x}_0 = R.\dot{\theta}_1.\vec{y}_1.\vec{x}_0 - R.\left(\dot{\theta}_1\right)\vec{y}_2.\vec{x}_0$ et à l'aide des figures planes on obtient :

$\vec{y}_1.\vec{x}_0 = -\sin\theta_1$

$\vec{y}_2.\vec{x}_0 = -\sin\left(\theta_1 + \theta_2\right) \rightarrow \vec{y}_2.\vec{x}_0 = -\sin\left(\theta_1 + \pi - 2.\theta_1\right) \rightarrow \vec{y}_2.\vec{x}_0 = -\sin\left(-\theta_1 + \pi\right) = -\sin\left(\theta_1\right)$

$\rightarrow \overrightarrow{V_{M,3/0}}.\vec{x}_0 = -R.\dot{\theta}_1.\sin\theta_1 + R.\dot{\theta}_1.\sin\theta_1 = 0 \rightarrow \boxed{\overrightarrow{V_{M,3/0}}.\vec{x}_0 = 0}$

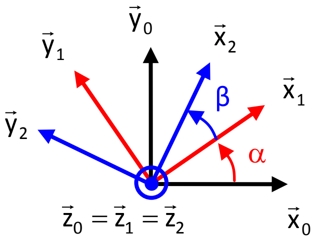
$\overrightarrow{V_{M,3/0}} = R.\dot{\theta}_1.\left(\vec{y}_1 - \vec{y}_2\right) \rightarrow \left\|\overrightarrow{V_{M,3/0}}\right\| = \sqrt{\overrightarrow{V_{M,3/0}}.\overrightarrow{V_{M,3/0}}} = R.\dot{\theta}_1.\sqrt{\left(\vec{y}_1 - \vec{y}_2\right)^2} = R.\dot{\theta}_1.\sqrt{2 - 2.\vec{y}_1.\vec{y}_2}$

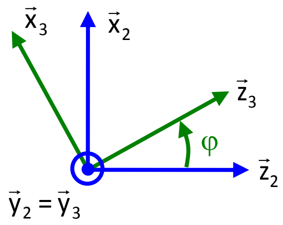
$\left\|\overrightarrow{V_{M,3/0}}\right\| = R.\dot{\theta}_1.\sqrt{2 - 2.\cos\theta_2} = R.\dot{\theta}_1.\sqrt{2 - 2.\cos\left(\pi - 2.\theta_1\right)} \rightarrow \left\|\overrightarrow{V_{M,3/0}}\right\| = R.\dot{\theta}_1.\sqrt{2 + 2.\cos\left(2.\theta_1\right)}$

Q.6. $\left\|\overrightarrow{V_{M,3/0}}\right\|$ est maxi pour $\theta_1 = 0 \rightarrow \left\|\overrightarrow{V_{M,3/0}}\right\| = 48 \times 0{,}08 \times \frac{2\pi}{60} \times 2 = 0{,}8$ cm/s < 2 cm/s → C.d.C.F ok.

Manège Magic Arms - Corrigé

Q.1.





$$\overrightarrow{\Omega_{1/0}} = \dot{\alpha}.\vec{z}_0$$
$$\overrightarrow{\Omega_{2/1}} = \dot{\beta}.\vec{z}_0$$
$$\overrightarrow{\Omega_{3/2}} = \dot{\varphi}.\vec{y}_2$$
$$\overrightarrow{\Omega_{2/0}} = \overrightarrow{\Omega_{21}} + \overrightarrow{\Omega_{10}} = \left(\dot{\alpha}+\dot{\beta}\right)\vec{z}_0$$
$$\overrightarrow{\Omega_{3/0}} = \overrightarrow{\Omega_{3/2}} + \overrightarrow{\Omega_{21}} + \overrightarrow{\Omega_{10}} = \left(\dot{\alpha}+\dot{\beta}\right)\vec{z}_0 + \dot{\varphi}.\vec{y}_2$$

Q.2. Le point P a une réalité physique sur le solide 3, on peut utiliser le calcul direct.

$$\overrightarrow{V_{P,3/0}} = \overrightarrow{V_{P/0}} = \frac{d}{dt}\overrightarrow{O_1P}\Big|_0 = \frac{d}{dt}\left(-l_1.\vec{y}_1 - l_2.\vec{y}_2 - R.\vec{z}_3\right)\Big|_0 = l_1.\dot{\alpha}.\vec{x}_1 + l_2.\left(\dot{\alpha}+\dot{\beta}\right)\vec{x}_2 - R.\frac{d}{dt}\left(\vec{z}_3\right)\Big|_0$$

Avec $\frac{d}{dt}\vec{z}_3\Big|_0 = \frac{d}{dt}\vec{z}_3\Big|_3 + \overrightarrow{\Omega_{3/0}} \wedge \vec{z}_3 = \left(\left(\dot{\alpha}+\dot{\beta}\right)\vec{z}_2 + \dot{\varphi}.\vec{y}_3\right) \wedge \vec{z}_3 = \left(\dot{\alpha}+\dot{\beta}\right)\sin\varphi.\vec{y}_2 + \dot{\varphi}.\vec{x}_3$

→ $\boxed{\overrightarrow{V_{P,3/0}} = l_1.\dot{\alpha}.\vec{x}_1 + l_2.\left(\dot{\alpha}+\dot{\beta}\right)\vec{x}_2 - R.\left(\dot{\alpha}+\dot{\beta}\right)\sin\varphi.\vec{y}_2 - R.\dot{\varphi}.\vec{x}_3}$

Q.3. Pour t [17-27] secondes on a les trois vitesses angulaires constantes → il y a donc 3 mouvements circulaires uniformes (accélérations nulles) :

$\boxed{\dot{\alpha} = 0{,}84 \text{ rad/s}}$ Loi du mouvement → α(t) = $\dot{\alpha}.t + cte_1$ et à t = 17s, on a α= 10,5 rad

$\boxed{\dot{\beta} = 0{,}94 \text{ rad/s}}$ Loi du mouvement → β(t) = $\dot{\beta}.t + cte_2$ et à t = 17s, on a β= 3,76 rad

$\boxed{\dot{\varphi} = -0{,}628 \text{ rad/s}}$ Loi du mouvement → φ(t) = $\dot{\varphi}.t + cte_3$ et à t = 17s, on a γ = –10,676 rad

→ α(17 s) =10,5 rad = $0{,}84 \times 17 + cte_1$ → $cte_1 = 10{,}5 - 0{,}84 \times 17 = -3{,}78$
→ β(17 s) = 3,76 rad = $0{,}94 \times 17 + cte_2$ → $cte_2 = 3{,}76 - 0{,}94 \times 17 = -12{,}22$
→ φ(17 s) = -10,676 rad = $-0{,}628 \times 17 + cte_3$ → $cte_3 = -10{,}676 + 0{,}628 \times 17 = 0$

Loi du mouvement → $\boxed{\alpha(t) = 0{,}84.t - 3{,}78}$
Loi du mouvement → $\boxed{\beta(t) = 0{,}94.t - 12{,}22}$
Loi du mouvement → $\boxed{\varphi(t) = -0{,}628.t}$

Q.4. Pour t_1=19,8 s on a :
α(19,8) = $\dot{\alpha}.t - 3{,}78 = 0{,}84 \times 19{,}8 - 3{,}78$ = 12,852 rad
β(19,8) = $\dot{\beta}.t - 12{,}22 = 0{,}94 \times 19{,}8 - 12{,}22$ = 6,392 rad
φ(19,8) = $\dot{\varphi}.t = -0{,}628 \times 19{,}8$ = –12,43 rad

Q.5. $\overrightarrow{V_{P,3/0}} = l_1.\dot{\alpha}.\vec{x}_1 + l_2.\left(\dot{\alpha}+\dot{\beta}\right)\vec{x}_2 - R.\left(\dot{\alpha}+\dot{\beta}\right)\sin\varphi.\vec{y}_2 - R.\dot{\varphi}.\vec{x}_3 = V_{x2}.\vec{x}_2 + V_{y2}.\vec{y}_2 + V_{z2}.\vec{z}_2$.
Avec :
$\vec{x}_1 = \cos\beta.\vec{x}_2 - \sin\beta.\vec{y}_2$
$\vec{x}_3 = -\sin\varphi.\vec{z}_2 + \cos\varphi.\vec{x}_2$

$$\overrightarrow{V_{P,3/0}} = l_1.\dot{\alpha}.(\cos\beta.\vec{x}_2 - \sin\beta.\vec{y}_2) + l_2.\left(\dot{\alpha}+\dot{\beta}\right)\vec{x}_2 - R.\left(\dot{\alpha}+\dot{\beta}\right)\sin\varphi.\vec{y}_2 - R.\dot{\varphi}.(-\sin\varphi.\vec{z}_2 + \cos\varphi.\vec{x}_2)$$

Soit : $\begin{cases} V_{x2} = l_1.\dot{\alpha}.\cos\beta + l_2.(\dot{\alpha}+\dot{\beta}) - R.\dot{\varphi}.\cos\varphi \\ V_{y2} = -l_1.\dot{\alpha}.\sin\beta - R.(\dot{\alpha}+\dot{\beta}).\sin\varphi \\ V_{z2} = +R.\dot{\varphi}.\sin\varphi \end{cases}$

A.N. : $\begin{cases} V_{x2} = 3{,}9\times 0{,}84\times\cos 6{,}392 + 2{,}87.(0{,}84+0{,}94) + 2{,}61\times 0{,}628.\cos(-12{,}43) \\ V_{y2} = -3{,}9\times 0{,}84.\sin 6{,}392 - 2{,}61.(0{,}84+0{,}94).\sin(-12{,}43) \\ V_{z2} = -2{,}61\times 0{,}628.\sin(-12{,}43) \end{cases}$

Soit $V_{x2} = 10$ m/s, $V_{y2} = -0{,}967$ m/s et $V_{z2} = -0{,}215$ m/s.

Q.6. $\overrightarrow{V_{P,3/0}} = l_1.\dot{\alpha}.\vec{x}_1 + l_2.(\dot{\alpha}+\dot{\beta}).\vec{x}_2 - R.(\dot{\alpha}+\dot{\beta}).\sin\varphi.\vec{y}_2 - R.\dot{\varphi}.\vec{x}_3$

$$\overrightarrow{\Gamma_{P,3/0}} = \frac{d}{dt}\overrightarrow{V_{P,3/0}}\Big|_0 = \frac{d}{dt}\left(l_1.\dot{\alpha}.\vec{x}_1 + l_2.(\dot{\alpha}+\dot{\beta}).\vec{x}_2 - R.(\dot{\alpha}+\dot{\beta}).\sin\varphi.\vec{y}_2 - R.\dot{\varphi}.\vec{x}_3\right)\Big|_0$$

$$\overrightarrow{\Gamma_{P,3/0}} = l_1.\dot{\alpha}.\frac{d}{dt}\vec{x}_1\Big|_0 + l_2.(\dot{\alpha}+\dot{\beta}).\frac{d}{dt}\vec{x}_2\Big|_0 - R.\dot{\varphi}.(\dot{\alpha}+\dot{\beta}).\cos\varphi.\vec{y}_2 - R.(\dot{\alpha}+\dot{\beta}).\sin\varphi l_2.\frac{d}{dt}\vec{y}_2\Big|_0 - R.\dot{\varphi}.\frac{d}{dt}\vec{x}_3\Big|_0$$

Avec :

$$\frac{d}{dt}\vec{x}_1\Big|_0 = \dot{\alpha}.\vec{y}_1$$

$$\frac{d}{dt}\vec{x}_2\Big|_0 = (\dot{\alpha}+\dot{\beta}).\vec{y}_2$$

$$\frac{d}{dt}\vec{y}_2\Big|_0 = -(\dot{\alpha}+\dot{\beta}).\vec{x}_2$$

$$\frac{d}{dt}\vec{x}_3\Big|_0 = \frac{d}{dt}\vec{x}_3\Big|_3 + \overrightarrow{\Omega_{3/0}}\wedge\vec{x}_3 = \left((\dot{\alpha}+\dot{\beta}).\vec{z}_2 + \dot{\varphi}.\vec{y}_3\right)\wedge\vec{x}_3 = (\dot{\alpha}+\dot{\beta}).\cos\varphi.\vec{y}_2 - \dot{\varphi}.\vec{z}_3$$

$$\boxed{\overrightarrow{\Gamma_{P,3/0}} = l_1.\dot{\alpha}^2.\vec{y}_1 + l_2.(\dot{\alpha}+\dot{\beta})^2.\vec{y}_2 - R.\dot{\varphi}.(\dot{\alpha}+\dot{\beta}).\cos\varphi.\vec{y}_2 + R.(\dot{\alpha}+\dot{\beta})^2.\sin\varphi.\vec{x}_2 - R.\dot{\varphi}.(\dot{\alpha}+\dot{\beta}).\cos\varphi.\vec{y}_2 + R.\dot{\varphi}^2.\vec{z}_3}$$

Q.7. Pour t_1=19,8 s, on a graphiquement $\left\|\overrightarrow{V_{P,3/0}}\right\| = 10$ m/s.

D'après Q.5. on a $\left\|\overrightarrow{V_{P,3/0}}\right\| = \sqrt{V_{x2}^2 + V_{y2}^2 + V_{z2}^2} = \sqrt{10^2 + 0{,}967^2 + 0{,}215^2}$ = 10,04 m/s

→ On retrouve les mêmes résultats.

Q.8. Graphiquement on a $\left\|\overrightarrow{\Gamma_{P,3/0}}\right\|_{max} = 17{,}7$ m/s^2 soit $\frac{17{,}7}{g} = \frac{17{,}7}{9{,}81} = 1{,}8$ g < 2,5.g → C.d.C.F. ok.