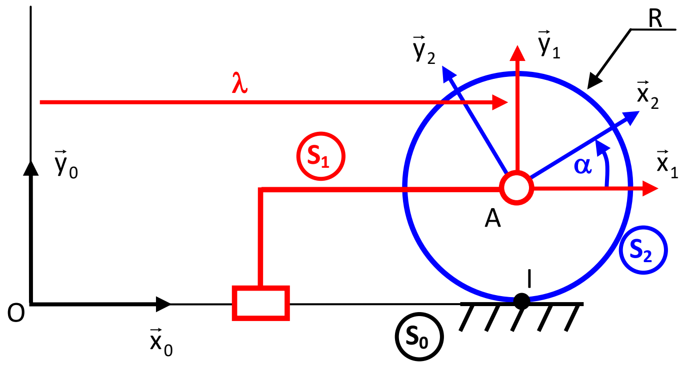
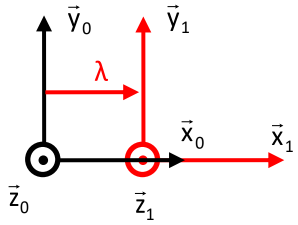
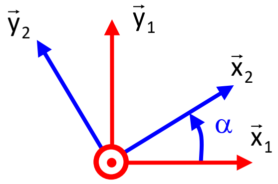
Roue de vélo sur le sol - Corrigé

Q.1.







Q.2. $\overrightarrow{V_{I,\,S_2/S_0}} = \vec{0}$

Q.3.

$$\overrightarrow{V_{I,\,S_2/S_0}} = \vec{0}$$

Nature du mouvement de 2/0 ? : Mouvement complexe.

→ On décompose en mouvements simples.
2/0 = 2/1 + 1/0

$$\overrightarrow{V_{I,\,S_2/S_0}} = \overrightarrow{V_{I,\,S_2/R_1}} + \overrightarrow{V_{I,\,S_1/R_0}}$$

Trajectoire du point I ∈ 2/0 (si RSG)

Nature du mouvement de 2/1? :
Rotation autour de l'axe (A, $\vec{z}_1$)

Champ des vitesses

$$\overrightarrow{V_{I,\,S_2/R_1}} = \overrightarrow{V_{A,\,S_2/R_1}} + \overrightarrow{IA} \wedge \overrightarrow{\Omega_{S_2/R_1}}$$

avec $\overrightarrow{V_{A,\,S_1/R_1}} = \vec{0}$ et

$$\overrightarrow{IA} \wedge \overrightarrow{\Omega_{S_2/R_1}} = R.\vec{y}_1 \wedge \dot{\alpha}.\vec{z}_0 = R.\dot{\alpha}.\vec{x}_0$$

$$\rightarrow \overrightarrow{V_{I,S_2/R_1}} = R.\dot{\alpha}.\vec{x}_0$$

Nature du mouvement de 1/0 ? :
Translation rectiligne suivant (O, $\vec{x}_0$)

Champ des vitesses

$$\overrightarrow{V_{I,\,S_1/R_0}} = \overrightarrow{V_{A,\,S_1/R_0}} = \left.\frac{d}{dt}\overrightarrow{OA}\right|_0$$

Le point est géométriquement bien défini on peut utiliser le calcul direct

$$\left.\frac{d}{dt}\overrightarrow{OA}\right|_0 = \left.\frac{d}{dt}\lambda.\vec{x}_0\right|_0 = \dot{\lambda}.\vec{x}_0$$

$$\rightarrow \overrightarrow{V_{I,S_1/R_0}} = \dot{\lambda}.\vec{x}_0$$

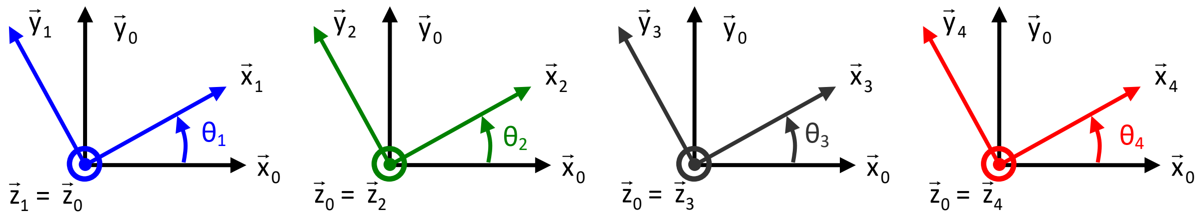
$$\rightarrow \overrightarrow{V_{I,\,S_2/S_0}} = \overrightarrow{V_{I,\,S_2/R_1}} + \overrightarrow{V_{I,\,S_1/R_0}} = \left(R.\dot{\alpha} + \dot{\lambda}\right)\vec{x}_0 = \vec{0} \;\rightarrow\; \boxed{R.\dot{\alpha} + \dot{\lambda} = 0}$$

Q.5. 50 km/h → $\dot{\lambda} = \frac{50000}{3600} = 13{,}4 \text{ m/s} \rightarrow \dot{\alpha} = -\frac{\dot{\lambda}}{R} = -\frac{13{,}4}{0{,}35} = -39{,}7 \text{ rd/s}$

$$\frac{\dot{\alpha}}{\dot{\theta}_{pedalier}} = \frac{Z_{pedalier}}{Z_{roue}} \rightarrow \dot{\theta}_{pedalier} = \frac{Z_{roue}}{Z_{pedalier}}.\dot{\alpha} = -\frac{14}{51}.39{,}7 = 10{,}9 \text{ rd/s}$$

$$\rightarrow \dot{\theta}_{pedalier} = \frac{2.\pi}{60}.N_{pedalier} \quad \text{soit } N_{pedalier} = \frac{60}{2.\pi}.10{,}9 = 104 \text{ tr/min}$$

Roulement à billes - Corrigé



$\overrightarrow{\Omega_{1/0}} = \dot{\theta}_1.\vec{z}_0$ $\quad$ $\overrightarrow{\Omega_{2/0}} = \dot{\theta}_2.\vec{z}_0$ $\quad$ $\overrightarrow{\Omega_{3/0}} = \dot{\theta}_3.\vec{z}_0$ $\quad$ $\overrightarrow{\Omega_{4/0}} = \dot{\theta}_4.\vec{z}_0$

Q.1. $\{C_{1/0}\} = {}_O\left\{\begin{matrix}\overrightarrow{\Omega_{1/0}} \\ \overrightarrow{V_{O,1/0}}\end{matrix}\right\} = {}_I\left\{\begin{matrix}\overrightarrow{\Omega_{1/0}} \\ \overrightarrow{V_{I,1/0}}\end{matrix}\right\}$ avec :

$\overrightarrow{\Omega_{1/0}} = \dot{\theta}_1.\vec{z}_0$

$\overrightarrow{V_{O,1/0}} = \vec{0}$

$\overrightarrow{V_{I,1/0}} = \overrightarrow{V_{O,1/0}} + \overrightarrow{IO} \wedge \overrightarrow{\Omega_{1/0}} = -r_1.\vec{i} \wedge \dot{\theta}_1.\vec{z}_0 = r_1.\dot{\theta}_1.\vec{j}$

$\rightarrow \overrightarrow{V_{I,1/0}} = r_1.\omega_1.\vec{j}$ $\;(\omega_k = \dot{\theta}_k \;\; (k = 1,2,3,4))$

$\{C_{2/0}\} = {}_O\left\{\begin{matrix}\overrightarrow{\Omega_{2/0}} \\ \overrightarrow{V_{O,2/0}}\end{matrix}\right\} = {}_J\left\{\begin{matrix}\overrightarrow{\Omega_{2/0}} \\ \overrightarrow{V_{J,2/0}}\end{matrix}\right\}$ avec :

$\overrightarrow{\Omega_{2/0}} = \dot{\theta}_2.\vec{z}_0$

$\overrightarrow{V_{O,2/0}} = \vec{0}$

$\overrightarrow{V_{J,2/0}} = \overrightarrow{V_{O,2/0}} + \overrightarrow{JO} \wedge \overrightarrow{\Omega_{2/0}} = -r_2.\vec{i} \wedge \dot{\theta}_2.\vec{z}_0 = r_2.\dot{\theta}_2.\vec{j}$

$\rightarrow \overrightarrow{V_{J,2/0}} = r_2.\omega_2.\vec{j}$ $\;(\omega_k = \dot{\theta}_k \;\; (k = 1,2,3,4))$

Q.2. RSG en I et J : $\overrightarrow{V_{I\in 3/1}} = \vec{0}$ et $\overrightarrow{V_{J\in 3/2}} = \vec{0}$

$\overrightarrow{V_{I\in 3/0}} = \overrightarrow{V_{I\in 3/1}} + \overrightarrow{V_{I\in 1/0}}$ (composition de mouvement) $\rightarrow \overrightarrow{V_{I\in 3/0}} = \overrightarrow{V_{I\in 1/0}} = r_1.\omega_1.\vec{j}$

$\overrightarrow{V_{J\in 3/0}} = \overrightarrow{V_{J\in 3/2}} + \overrightarrow{V_{J\in 2/0}}$ (composition de mouvement) $\rightarrow \overrightarrow{V_{J\in 3/0}} = \overrightarrow{V_{J\in 2/0}} = r_2.\omega_2.\vec{j}$

Q.3. $\overrightarrow{V_{I,3/0}} = \overrightarrow{V_{J,3/0}} + \overrightarrow{IJ} \wedge \overrightarrow{\Omega_{3/0}}$ (champ des vitesses)

$\rightarrow r_1.\omega_1.\vec{j} = r_2.\omega_2.\vec{j} + (r_2 - r_1).\vec{i} \wedge \omega_3.\vec{z}_0$

$\rightarrow r_1.\omega_1.\vec{j} = r_2.\omega_2.\vec{j} - (r_2 - r_1)\omega_3.\vec{j}$

$\rightarrow \boxed{\omega_3 = \dfrac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1}}$

Q.4. $\overrightarrow{V_{G,3/0}} = \overrightarrow{V_{I,3/0}} + \overrightarrow{GI} \wedge \overrightarrow{\Omega_{3/0}}$ (champ des vitesses)

$\rightarrow \overrightarrow{V_{G,3/0}} = r_1.\omega_1.\vec{j} - \dfrac{r_2 - r_1}{2}.\vec{i} \wedge \dfrac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1}.\vec{z}_0 = r_1.\omega_1.\vec{j} + \dfrac{r_2.\omega_2 - r_1.\omega_1}{2}.\vec{j}$

$\rightarrow \boxed{\overrightarrow{V_{G,3/0}} = \dfrac{r_2.\omega_2 + r_1.\omega_1}{2}.\vec{j}}$

Q.5. $\overrightarrow{V_{C,3/4}} = \overrightarrow{V_{G,3/4}} + \overrightarrow{CG} \wedge \overrightarrow{\Omega_{3/4}}$

On considère que la bille est en liaison pivot d'axe $(G, \vec{z}_0)$ avec la cage 4 $\rightarrow \overrightarrow{V_{G,3/4}} = \vec{0}$

Avec $\overrightarrow{CG} = -\dfrac{1}{2}.(r_2 - r_1).\vec{j}$ et $\overrightarrow{\Omega_{3/4}} = \overrightarrow{\Omega_{3/0}} - \overrightarrow{\Omega_{4/0}} = \omega_3.\vec{z}_0 - \omega_4.\vec{z}_0$ et $\omega_3 = \dfrac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1}$ (Q.3.)

Calcul de ω_4 :

$\overrightarrow{V_{G,3/4}} = \overrightarrow{V_{G,3/0}} - \overrightarrow{V_{G,4/0}} = \vec{0}$ (composition de mouvement)

Avec $\overrightarrow{V_{G,3/0}} = \dfrac{r_2.\omega_2 + r_1.\omega_1}{2}.\vec{j}$ (Q.4.)

→ $\overrightarrow{V_{G,4/0}} = \overrightarrow{V_{O,4/0}} + \overrightarrow{GO} \wedge \overrightarrow{\Omega_{4/0}} = -\left(r_1 + \frac{r_2 - r_1}{2}\right).\vec{i} \wedge \omega_4.\vec{z}_0 = \frac{r_2 + r_1}{2}.\omega_4.\vec{j}$

→ $\overrightarrow{V_{G,3/4}} = \frac{r_2.\omega_2 + r_1.\omega_1}{2}.\vec{j} - \frac{r_2 + r_1}{2}.\omega_4.\vec{j} = \vec{0}$

→ $\omega_4 = \frac{r_2.\omega_2 + r_1.\omega_1}{r_2 + r_1}$

→ $\overrightarrow{\Omega_{3/4}} = \overrightarrow{\Omega_{3/0}} - \overrightarrow{\Omega_{4/0}} = \frac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1}.\vec{z}_0 - \frac{r_2.\omega_2 + r_1.\omega_1}{r_2 + r_1}.\vec{z}_0$

D'où : $\overrightarrow{V_{C,3/4}} = -\frac{1}{2}.(r_2 - r_1).\vec{j} \wedge \left(\frac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1}.\vec{z}_0 - \frac{r_2.\omega_2 + r_1.\omega_1}{r_2 + r_1}.\vec{z}_0\right)$

→ $\overrightarrow{V_{C,3/4}} = -\frac{1}{2}.(r_2 - r_1).\left(\frac{r_2.\omega_2 - r_1.\omega_1}{r_2 - r_1} - \frac{r_2.\omega_2 + r_1.\omega_1}{r_2 + r_1}\right).\vec{i}$

→ $\overrightarrow{V_{C,3/4}} = -\frac{1}{2}.\frac{(r_2.\omega_2 - r_1.\omega_1).(r_2 + .r_1) - (r_2.\omega_2 + r_1.\omega_1).(r_2 - r_1)}{r_2 + r_1}.\vec{i}$

→ $\boxed{\overrightarrow{V_{C,3/4}} = \frac{r_2.r_1.(\omega_1 - \omega_2).}{r_2 + r_1}.\vec{i}}$