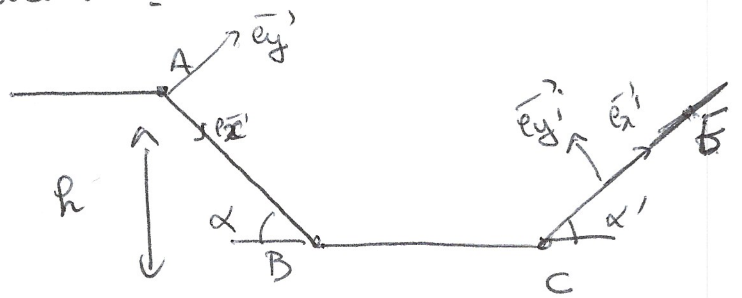
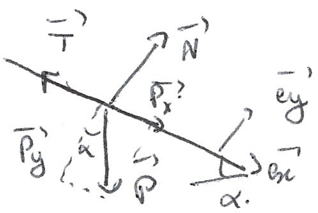
Correction DS 4 06/01/22.

Cours.

Application.



a] TEC $\quad \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = W_{A\to B}(\vec{P}) + \underbrace{W_{A\to B}(\vec{N})}_{0 \text{ car } \vec{N} \perp \vec{d\Omega}} + W_{A\to B}(\vec{T})$.



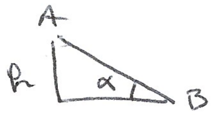
$W_{A\to B}(\vec{P}) = +mgh$.

Calcul de $\vec{N}$ et $\vec{T}$ avec $\|\vec{T}\| = f \cdot \|\vec{N}\|$.

$\vec{T} = -\|\vec{T}\| \vec{e_x} = -T\vec{e_x}$ et le PFD projeté suivant $\vec{e_y}'$ donne $\quad \vec{P_y} + \vec{N} = \vec{0}$

$(-mg\cos\alpha + N)\,\vec{e_y}' = 0 \Rightarrow N = mg\cos\alpha$ et $T = f \cdot mg\cos\alpha$.

$W_{A\to B}(\vec{T}) = -T \cdot AB$



$\sin\alpha = \frac{h}{AB} \Rightarrow AB = \frac{h}{\sin\alpha}$.

d'où $\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = mgh - fmgh \frac{\cos\alpha}{\sin\alpha} = mgh\left(1 - \frac{f}{\operatorname{tg}\alpha}\right)$.

$\left(1 - \frac{f}{\operatorname{tg}\alpha}\right) = \frac{v_B^2 - v_A^2}{2 \cdot g \cdot h}$. $\Rightarrow \frac{f}{\operatorname{tg}\alpha} = 1 - \frac{(v_B^2 - v_A^2)}{2gh} \Rightarrow$

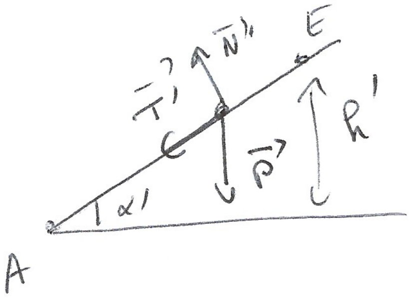
$$\boxed{\operatorname{tg}\alpha = \frac{f}{1 - \frac{(v_B^2 - v_A^2)}{2gh}}}$$

AN $\boxed{\alpha = 4{,}7°}$.

2°) $\frac{1}{2} m v_E^2 - \frac{1}{2} m v_C^2 = W_{C\to E}(\vec{P}) + \underbrace{W_{C\to E}(\vec{N}')}_{0} + W_{C\to E}(\vec{T}')$.

De la même façon : $\|\vec{T}'\| = f\|\vec{N}'\| = f \cdot mg\cos\alpha'$

(1)



$AE = D$ et $v_E = 0$

$W_{C\to D}(\vec{P'}) = -mgh'$ et $\sin\alpha' = \dfrac{h'}{D}$

$= -mgD\cdot\sin\alpha'$

$W_{C\to D}(\vec{T'}) = \ominus f mg\cos\alpha' \cdot D$.

d'où $-\frac{1}{2} m v_B^2 = -mgD\sin\alpha' - f mg\cos\alpha' \cdot D$.

$\dfrac{v_B^2}{2} = g[\sin\alpha' + f\cos\alpha']\cdot D \Rightarrow \boxed{D = \dfrac{v_B^2}{2g(\sin\alpha' + f\cos\alpha')}}$

<u>AN</u>

$\alpha' = \alpha = 4{,}7°$ $\quad D = 4{,}4\,m$.

$\alpha' = 10°$ $\quad D = 2{,}2\,m$.

b] Le colis reste immobile en E si $\|\vec{T}\| < f\cdot\|\vec{N}\|$.

avec $\|\vec{N}\| = mg\cos\alpha'$ ⇒ $mg\sin\alpha' < f\cdot mg\cos\alpha$

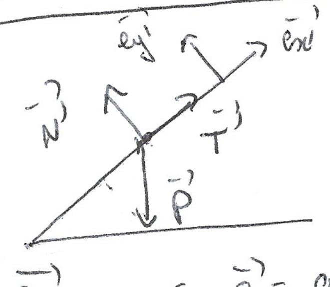
$\underbrace{\|\vec{T}\| = mg\sin\alpha'}_{\text{projection de la relation } \Sigma\vec{F}=0 \text{ suivant } \vec{ex'}}$ → $\boxed{\tan\alpha' < f}$.

<u>AN</u> $\alpha' = 4{,}7°$. $\tan\alpha' = 0{,}08$ $\quad f = 0{,}1$ Condition vérifiée.

$\alpha' = 10°$ $\quad \tan\alpha' = 0{,}17 > f$ ⇒ Condition non vérifiée.

→ le colis redescend.

<u>Etude de la descente.</u>



$\|\vec{T'}\| = f\cdot mg\cos\alpha'$ et $\vec{T'} = +fmg\cos\alpha'\,\vec{ex'}$

$\vec{P'} = -mg\sin\alpha'\,\vec{ex'} - mg\cos\alpha'\,\vec{ey'}$.

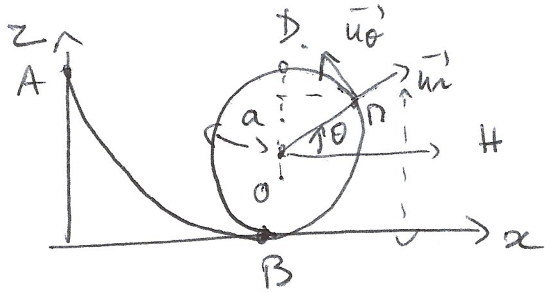
$m\vec{a} = \vec{P'} + \vec{T'} + \vec{N'}$ donc en projection suivant $\vec{ex'}$ avec $\vec{a} = a\,\vec{ex'}$

$ma = -mg\sin\alpha' + fmg\cos\alpha' \Rightarrow$

$a = -g\cos\alpha'[\tan\alpha' - f] < 0$.

$a = -0{,}75\ m\,s^{-2}$

EX1 Conditions pour effectuer un Looping -



1°] TEC $\frac{1}{2}mv_B^2 - \underbrace{\frac{1}{2}mv_A^2}_{0} = mgh \rightarrow v_B = \sqrt{2gh}$

ou

TEM $\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2 + 0 \Rightarrow v_B = \sqrt{2gh}$.

2°) $\frac{1}{2}mv_M^2 - \frac{1}{2}mv_B^2 = W_{B\to M}(\vec{P}) + \underbrace{W_{B\to M}(\vec{R})}_{0}$ car $\vec{R} \perp d\vec{M}$ en l'absence de frottements -

$W_{B\to M}(\vec{P}) = -mgH = -mg(a + a\sin\theta)$.

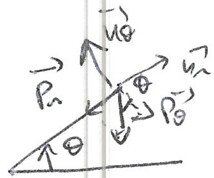
d'où $v_M^2 = v_B^2 - 2ga(1+\sin\theta)$

$v_M^2 = 2gh - 2ga(1+\sin\theta) \Rightarrow \boxed{v_M = \sqrt{2gh - 2ga(1+\sin\theta)}}$

3°) La bille effectue le looping si $v_D \neq 0$ et $v_D > 0$.

en D $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1 \rightarrow v_D = \sqrt{2gh - 4ga} \Rightarrow$

$\boxed{h > 2a.} \rightarrow \boxed{h_{min} = 2a.}$



4°] $\vec{P} + \vec{R} = m\vec{a}$

$\vec{R} = -R\vec{u_r}$

$\vec{P} = -mg\sin\theta\,\vec{u_r} - mg\cos\theta\,\vec{u_\theta}$

$\vec{OM} = a\vec{u_r} \Rightarrow \vec{v_M} = a\dot{\theta}\vec{u_\theta} \rightarrow \vec{a_M} = -a\dot{\theta}^2\vec{u_r} + a\ddot{\theta}\vec{u_\theta} = -\frac{v_M^2}{a}\vec{u_r} + a\ddot{\theta}\vec{u_\theta}$

d'où $m\vec{a_M} = \vec{P} + \vec{R}$ projetée sur

$\vec{u_r}$: $-\frac{mv_M^2}{a} = -mg\sin\theta - R \Rightarrow R = \frac{mv_M^2}{a} - mg\sin\theta$.

$\vec{u_\theta}$; $ma\ddot{\theta} = -mg\cos\theta$

$= \frac{m}{a}\left[2gh - 2ga(1+\sin\theta)\right] - mg\sin\theta$

$= 2mg\frac{h}{a} - 2mg - 2mg\sin\theta - mg\sin\theta$

$\Rightarrow R(\theta) = 2mg\frac{h}{a} - 2mg - 3mg\sin\theta$

$R(\theta) = mg\left[\frac{2h}{a} - 2 - 3\sin\theta\right]$.

R est min si $\sin\theta$ est max $\Rightarrow$ $\sin\theta = 1$ $\rightarrow$ $\theta = \frac{\pi}{2}$ donc en D.

$R_{min}(D) = mg\left[\frac{2h}{a} - 5\right]$.

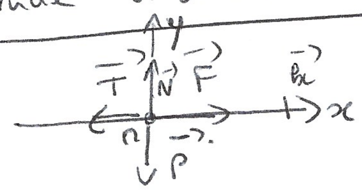
La bille est en contact en D si $R_{min}(D) > 0$ $\Rightarrow$ $\boxed{h > \underbrace{\frac{5}{2}a}_{h_{min2}}}$.

d] $h_{min2} > h_{min1}$.

si $h > h_{min2}$ alors $h > h_{min1}$. $\Rightarrow$ $\boxed{h_{min} = h_{min2} = \frac{5}{2}a}$.

Ex 2 Etude d'un traîneau.

1.1.



M immobile $\Rightarrow$ $\underbrace{\vec{P} + \vec{N}}_{= \vec{0}} + \vec{T} + \vec{F} = \vec{0}$ et $\|\vec{T}\| < f_0 \cdot \|\vec{N}\|$.

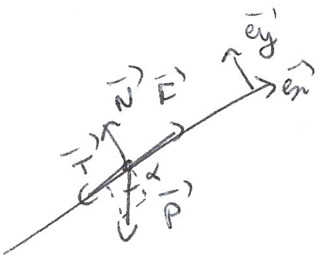
$\vec{F} = F\vec{e_x}$ $\quad \vec{T} = -T\vec{e_x}$ $\quad \vec{P} = -mg\vec{e_y}$ $\quad \vec{N} = N\vec{e_y}$ $\quad T > 0$ $\quad N > 0$.

en proj/$\vec{e_x}$ $F - T = 0$ $\rightarrow$ $T = F$

proj/$\vec{e_y}$ $N - mg = 0$ $\rightarrow$ $N = mg$.

d'où $T < f_0 \cdot N$ $\Rightarrow$ $F < f_0 mg$. $\Rightarrow$ $\boxed{F_{min} = f_0 mg}$.

1.2]



$\vec{T} = -T\vec{e_x}$

$\vec{F} = F\vec{e_x}$

$\vec{P} = -mg\sin\alpha\,\vec{e_x} - mg\cos\alpha\,\vec{e_y}$

$\vec{N} = N\vec{e_y}$

$\vec{P} + \vec{T} + \vec{F} + \vec{N} = \vec{0}$

proj/$\vec{e_x}$ $-mg\sin\alpha - T + F = 0$

/$\vec{e_y}$ $-mg\cos\alpha + N = 0$

$\rightarrow$ $T = F - mg\sin\alpha$

$N = mg\cos\alpha$

et $T < f \cdot N$.

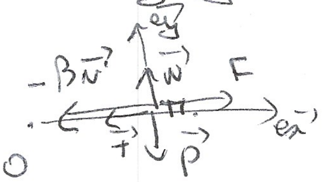
$F - mg\sin\alpha < f mg$

$\Rightarrow$ $F < mg\underbrace{f\left[\cos\alpha + \frac{\sin\alpha}{f}\right]}_{f'}$. avec $\boxed{f' = f\cos\alpha + \sin\alpha}$

3] La traînée dominante $\Rightarrow F > f mg$ et $\|\vec{T}\| = f \cdot \|\vec{N}\|$.

$\vec{T}$ opposé à $\vec{v}$.

$$m\vec{a} = \vec{P} + \vec{T} + \vec{N} - \beta\vec{v} + \vec{F}$$



$\vec{T} = -\|\vec{T}\|\,\vec{e_x}$

$\vec{N} = N\vec{e_y}$

$\vec{F} = F\vec{e_x}$

$\vec{OM} = x\,\vec{e_x}$

$\vec{v} = \dot{x}\,\vec{e_x} = v\,\vec{e_x}$ $\quad \vec{a} = \dot{v}\,\vec{e_x}$.

PFD projeté suivant $\vec{e_x}$: $m\dot{v} = -f\cdot\|\vec{N}\| - \beta v + F$

$\vec{e_y}$: $N - mg = 0 \Rightarrow N = \|\vec{N}\| = mg$.

$$m\dot{v} + \beta v = -f mg + F \rightarrow \dot{v} + \frac{\beta}{m} v = -f\cdot g + \frac{F}{m}. \qquad \tau = \frac{m}{\beta}.$$

$$v(t) = A e^{-\frac{t}{\tau}} + \underbrace{(F - f mg)/\beta}_{SP} \qquad t > 10\tau. \qquad v(t) \rightarrow \boxed{\frac{F - f\,mg}{\beta} = v_{lim}}.$$

$$v(0) = 0 \Rightarrow \boxed{v(t) = \frac{F - f mg}{\beta}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)}$$

5) AN

$\boxed{v_e = \dfrac{F - f\cdot mg}{\beta}}$

$m = 5{,}0\cdot 10^{1}$ kg.

$f = 5\cdot 10^{-2}$

$g = 9{,}8$ m s^{-2}

$v(t_1) = 95\%\, v_{lim} \rightarrow v_{lim}\left(1 - \exp\left(-\frac{t_1}{\tau}\right)\right) = 0{,}95\, v_{lim}$.

$0{,}05\,\dfrac{v_{lim}}{v_{lim}} = \exp\left(-\frac{t_1}{\tau}\right) \rightarrow \boxed{t_1 = -\tau \ln 0{,}05}$ [redacted].

$v_{lim} = 3$ m s^{-1}

$t_1 = 5$ s

$\tau = +\dfrac{m}{\beta} \rightarrow -\dfrac{m}{\beta}\ln 0{,}05$ [redacted] $= t_1 \rightarrow \beta = -\dfrac{m \ln 0{,}05}{t_1}$

$\Rightarrow \boxed{\beta = 3\cdot 10^{2}\ \text{kg s}^{-2}}$. $\qquad \boxed{F = \beta v_{lim} + f\cdot mg = 1{,}14\cdot 10^{3}\ \text{N}}$.

6] $\dfrac{dE_c}{d} = \underbrace{\delta W_{\vec{P}}}_{=0} + \underbrace{\delta W_{\vec{N}}}_{=0} + \delta W_{\vec{T}} + \delta W(-\beta v) + \delta W(\vec{F}) = (\vec{T} - \beta\vec{v} + \vec{F})\,\vec{v}\,dt$

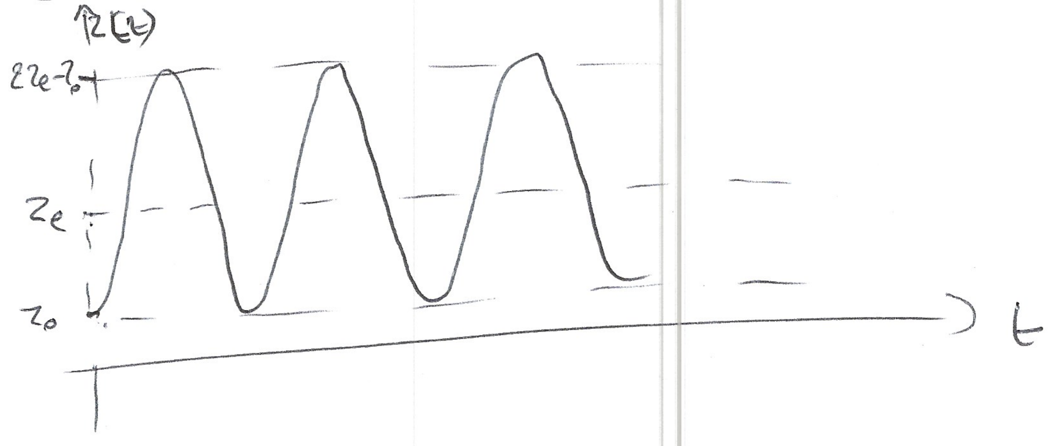
$\dfrac{dE_c}{dt} = (\vec{T} - \beta\vec{v} + \vec{F})\,\vec{v} \Rightarrow \frac{1}{\cancel{2}}\cancel{m}\cdot\cancel{2}\ddot{x}\dot{x} = -f mg\,\dot{x} - \beta\dot{x}^2 + F\dot{x}$,

$\ddot{x} = -fg - \dfrac{\beta\dot{x}}{m} + \dfrac{F}{m} \Rightarrow \boxed{\ddot{x} + \dfrac{\beta}{m}\dot{x} = \dfrac{F}{m} - f\!\!}$

6°] $z(t)$ centrée sur $z_e = \langle z(t) \rangle$

$z_{max} = 2z_e - z_0$

$z_{min} = z_0$.



Partie 2. Suspension avec amortissement.

7] $\vec{F} = -h\vec{v} \Rightarrow [h] = \frac{[F]}{[v]} = \frac{MLT^{-2}}{LT^{-1}} = MT^{-1} \Rightarrow \boxed{h \text{ en } kg\,s^{-1}}$

8]. $\vec{P} = -mg\vec{e_z}$ $\quad \vec{F} = -h\dot{z}\vec{e_z}$.

$\vec{T} = -k(z-l_0)\vec{e_z}$

A l'eq. $\vec{P} + \vec{T} + \underset{\substack{\| \\ 0 \text{ car } \dot{z}=0}}{\vec{F}} = 0 \rightarrow \vec{P} + \vec{T} = \vec{0}$ $\hat{n}$ pentre d'eq.

9] PFD $\vec{P} + \vec{T} + \vec{F} = m\vec{a}$.

proj/$\vec{e_z}$ $-mg - k(z-l_0) - h\dot{z} = m\ddot{z} \rightarrow m\ddot{z} + h\dot{z} + kz = k\left(l_0 - \frac{mg}{k}\right)$

$m\ddot{z} + h\dot{z} + kz = kz_e \rightarrow \ddot{z} + \underbrace{\frac{h}{m}}_{2\lambda\omega_0}\dot{z} + \underbrace{\frac{k}{m}}_{\omega_0^2}z = \underbrace{\frac{k}{m}}_{\omega_0^2}z_e$.

$\omega_0 = \sqrt{\frac{k}{m}}$

$2\lambda\omega_0 = \frac{h}{m} \rightarrow \lambda = \frac{h}{2m\omega_0} = \frac{h}{2m\sqrt{\frac{k}{m}}} \rightarrow \boxed{\lambda = \frac{h}{2\sqrt{mk}}}$

On retrouve l'eq. diff par une méthode énergétique

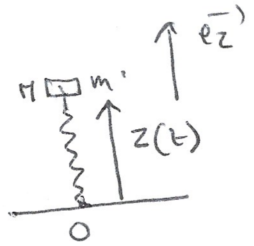
$\frac{dE_m}{dt} = \vec{F}.\vec{v} \Rightarrow \frac{2}{2}m\ddot{z}\dot{z} + mg\dot{z} + \frac{2k}{2}\dot{z}(z-l_0) = -h\dot{z}^2$.

d'où $\boxed{m\ddot{z} + h\dot{z} + kz = kz_e}$.

Ex3.

$1^{ère}$ partie :

1)



$\vec{P} = -mg\,\vec{e_z}$.

$\vec{T} = -k(z-l_0)\,\vec{e_z}$. (dirigé vers la position à vide).

2) PFD $\vec{P}+\vec{T} = m\vec{a}$. A l'équilibre $\begin{cases}\vec{v}=0 \\ \forall t\end{cases} \Rightarrow \vec{a}=0$

$\rightarrow \vec{P}+\vec{T}=\vec{0}$

proj$/\vec{e_z}$ $\quad -mg - k(z_e - l_0) = 0 \rightarrow \boxed{z_e = l_0 - \frac{mg}{k}}$

3) $\vec{OM} = z(t)\,\vec{e_z}$ $\longrightarrow$ PFD proj$/\vec{e_z}$

$\vec{a} = \ddot{z}(t)\,\vec{e_z}$ $\quad -mg - k(z-l_0) = m\ddot{z} \Rightarrow$

$m\ddot{z} + kz = -mg + kl_0 = k\left(l_0 - \frac{mg}{k}\right) = k\,z_e$.

$\boxed{\ddot{z} + \frac{k}{m}z = \frac{k}{m}z_e}$. et $\boxed{z(t) = A\cos(\omega_0 t + \varphi) + z_e}$.

Par une méthode énergétique : $dE_c = \delta W_{\vec{P}} + \delta W_{\vec{T}} = -d(E_{ppes} + E_{pel})$

$\Rightarrow d(E_c + E_p) = 0 \longrightarrow E_m = cste$ ~~avec~~ $E_m = \frac{1}{2}m\dot{z}^2 + mgz + \frac{1}{2}k(z-l_0)^2$

donc $\frac{dE_m}{dt} = 0 \Rightarrow \frac{2}{2}m\ddot{z}\cdot\dot{z} + mg\dot{z} + \frac{2k}{2}\dot{z}(z-l_0) = 0$.

$m\ddot{z} + mg + k(z-l_0) = 0 \Rightarrow \boxed{m\ddot{z} + kz = k\left(l_0 - \frac{mg}{k}\right)}$. (On retrouve la même éq. différentielle)

4) $\omega_0^2 = \frac{k}{m}$ et $T_0 = \frac{2\pi}{\omega_0} = \boxed{2\pi\sqrt{\frac{m}{k}} = T_0}$. AN $\boxed{\begin{array}{l}\omega_0 = 10\,rd/s \\ T_0 = 0{,}63\,s\end{array}}$.

$\boxed{\omega_0 = \sqrt{\frac{k}{m}}}$

5) $z(t=0) = z_0 \longrightarrow z_0 = A\cos\varphi + z_e$. $\Rightarrow A = z_0 - z_e$.

$\dot{z}(t=0) = 0 \qquad 0 = -A\omega_0\sin\varphi \rightarrow \varphi = 0$

$\boxed{z(t) = \underbrace{(z_0 - z_e)}_{<0}\cos(\omega_0 t) + z_e} = \underbrace{(z_e - z_0)}_{>0}\cos\left(\omega_0 t + \frac{\pi}{2}\right) + z_e$

10]. $\ddot z + 2\lambda\omega_0 \dot z + \omega_0^2 z = \omega_0^2 z_e$.

$z(t) = SG_0(t) + z_e$.

<u>Calcul de $SG_0(t)$</u> : $\ddot z + 2\lambda\omega_0 \dot z + \omega_0^2 z = 0$.

e^{rt} solution $\quad r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$\Delta = 4\lambda^2\omega_0^2 - 4\omega_0^2 = 4\omega_0^2(\lambda^2 - 1)$.

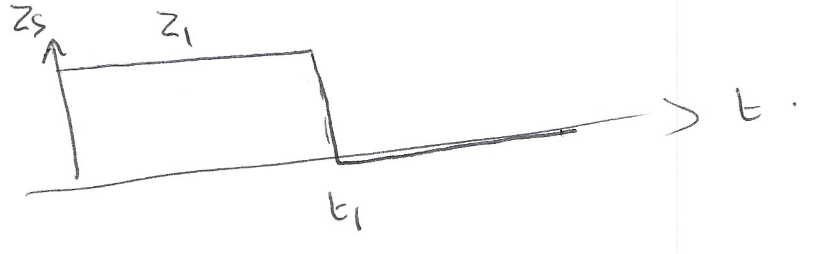
Regime pseudo periodique $\lambda < 1$
critique $\lambda = 1$
aperiodique $\lambda > 1$
avec $\lambda = \dfrac{h}{2\sqrt{km}}$.

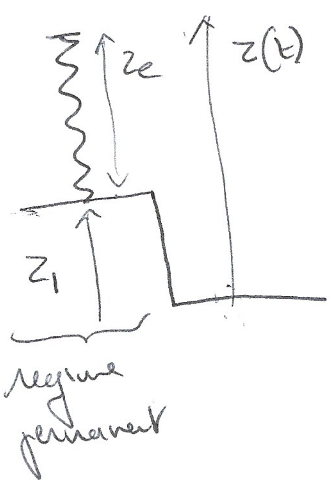
$\lambda < 1 \Rightarrow \boxed{h < 2\sqrt{km}}$ regime pseudo périodique.

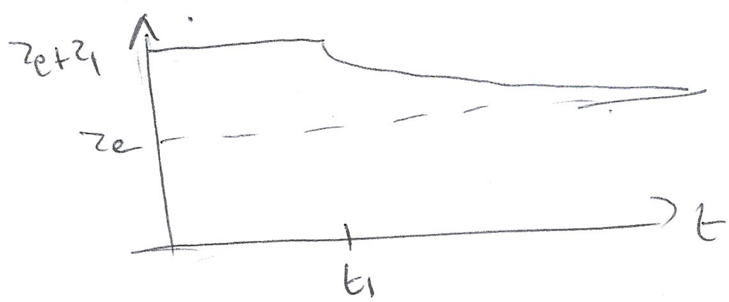
11-1] Vehicule à vide $\Rightarrow$ regime critique $h = 2\sqrt{km}$.
en charge $\Rightarrow$ $m \nearrow$ avec $2\sqrt{km} > h$ regime pseudo periodique.

11-2] Regime pseudo periodique $\to$ nombreuses oscillations (A eviter).
Il faut choisir $h > 2\sqrt{km}$ $\quad m =$ masse voit.

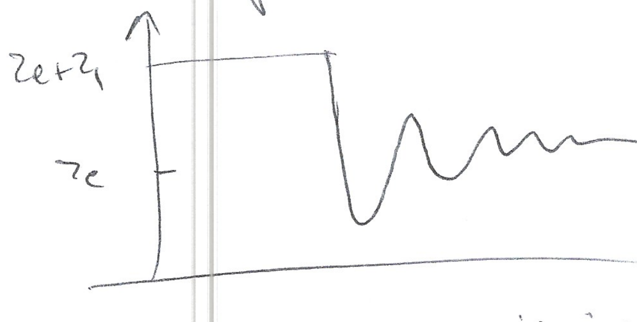
12].







Regime aperiodique.



Regime pseudoperiodique (A eviter)