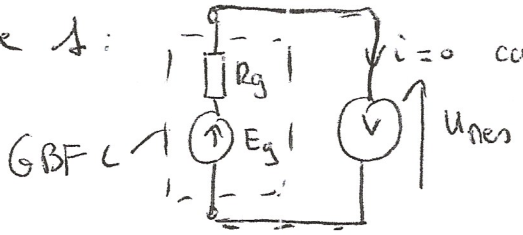
Exercice 1

Correction DS5 27/01/23.

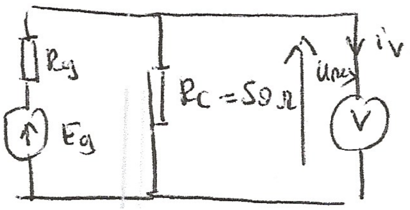
1) Mesure 1 :



$i = 0$ car $R_V \to \infty$

$\boxed{U_{mes_1} = E_g = 6\,V}$

Mesure 2

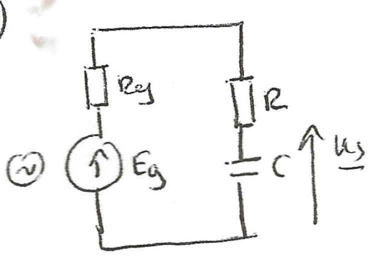


$i_V = 0$ car $R_V = \infty$

$U_{mes_2} = \frac{R_C}{R_C + R_g} \cdot E_g$.

$\Rightarrow R_C + R_g = \frac{R_C \cdot U_{mes1}}{U_{mes2}} \Rightarrow R_g = \frac{R_C \cdot U_{mes1}}{U_{mes2}} - R_C = 50\left(\frac{6}{3} - 1\right) = \boxed{50\,\Omega = R_g}$

2)



$\underline{Z_C} = R + \frac{1}{jC\omega}$ $\quad \underline{i} = \frac{E_g}{R_g + R + \frac{1}{jC\omega}} \Rightarrow |\underline{i}| = \frac{E_g}{\sqrt{(R_g+R)^2 + \frac{1}{C^2\omega^2}}}$

$\underline{i} \to \frac{E_g}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}$ si $R_g \ll R$

Impédance de charge minimale $\omega \to \infty$

3)

$\underline{u_s} = \frac{\frac{1}{jC\omega} \cdot E_g}{\frac{1}{jC\omega} + R} = \frac{E_g}{1 + jRC\omega}$

$\omega \to 0 \quad \underline{u_s} \to E$ Filtre passe bas.

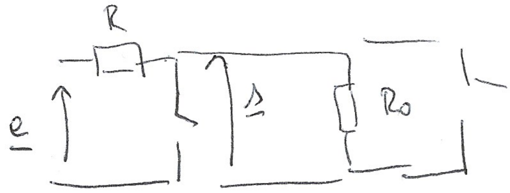
$\omega \to \infty \quad \underline{u_s} \to 0$

ω_c est défini par $|\underline{u_s}(\omega_c)| = \frac{|u_s|_{max}}{\sqrt{2}} = \frac{E_g}{\sqrt{2}} \to \boxed{\omega_c = \frac{1}{RC}}$.

AN $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi\, 4{,}7 \cdot 10^3 \cdot 22 \cdot 10^{-9}} = \frac{10^6}{\underbrace{2\pi \cdot 4{,}7 \times 22}_{6 \quad 30}} = \frac{10^6}{660} = 0{,}0015 \cdot 10^6$

$\boxed{\simeq 1{,}5\,kHz = f_c}$.

4°) à BF



Division de tension $\frac{\underline{s}}{\underline{e}} = \frac{R_0}{R + R_0}$.

5°) $\underline{Y_{eq}} = \frac{1}{R_0} + j(C_0 + C)\omega$. $\quad \underline{s} = \underline{Z} \cdot \underline{i} \to \underline{i} = \underline{Y} \cdot \underline{s}$

$\Rightarrow \varphi_s = \varphi_i - \varphi_Y \Rightarrow \Delta\varphi_{s/i} = \varphi_s - \varphi_i = -\varphi_Y = -\arctan R_0 (C + C_0)\omega \to 0$ lorsque $\omega \to 0$.

6°) $\underline{H} = \frac{Z_{eq}}{Z_{eq} + R} = \frac{1}{R Y_{eq} + 1} = \frac{1}{R\left[\frac{1}{R_0} + j(C_0 + C)\omega\right] + 1} = \frac{1}{1 + \frac{R}{R_0} + jR(C_0 + C)\omega}$

$$\underline{H} = \frac{R_0/R+R_0}{1+j\frac{R_0 R}{R_0+R}(C_0+C)\omega} \Rightarrow H'_0 = \frac{R_0}{R+R_0} \simeq 1 \simeq H_0$$

$$\omega'_c \; \frac{1}{\frac{R_0 R}{R+R_0}(C_0+C)\omega} \simeq \omega_c.$$

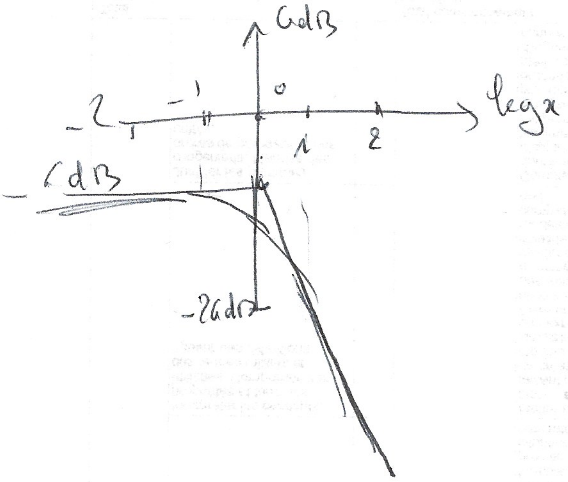
Filtre et oscillo adaptés.

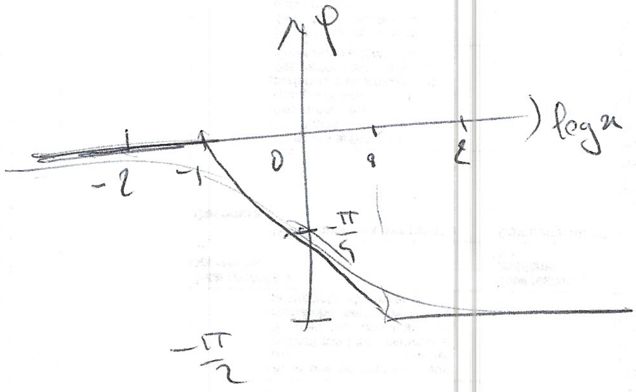
8°) $H = \frac{H_0}{1+jx} \xrightarrow{BF} H_0 \begin{cases} GdB \to Y_{BF} = 20 \log H_0 = -6dB \\ \varphi \to 0 \end{cases}$

$\xrightarrow{HF} \frac{H_0}{jx} \begin{cases} GdB \to Y_{HF} = 20 \log H_0 - 20 \log x \\ \varphi \to -\frac{\pi}{2}. \end{cases}$

$Y_{BF} = Y_{HF} \leftrightarrow I \begin{vmatrix} x=0 \\ 20 \log H_0. \end{vmatrix}$

$GdB(\omega_c) = -9dB.$

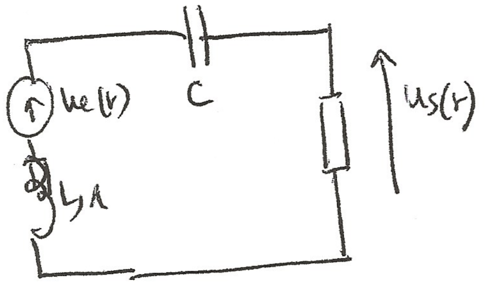




10°) $e(t) = E_0 \cdot H_0 \cos(0{,}01\omega_0 t) + \frac{E_0}{100} \cos\left(10\omega_0 t - \frac{\pi}{2}\right) + \frac{E_0}{1000}\left(100\omega_0 t - \frac{\pi}{2}\right).$

Amplitude/10 amplitude/100

<u>Partie 1</u> <u>002</u>.



1°) $u_s(t) = Ri(t)$

BF ⊣⊢ ⇔ open ⇒ $i=0$ ⇒ $u_s = 0$ $= Ri$

HF ⌇ ⇔ open ⇒ $i=0$ ⇒ $u_s = 0$. $= Ri$

Filtre passe-bande.

2°) $u_e(t) = E\cos\omega t \to \underline{u_e} = E$

$u_s(t) = U_s\cos(\omega t + \varphi) \Rightarrow \underline{u_s} = U_s e^{j\varphi}$.

3°) $$\underline{H} = \frac{R}{R+r+jL\omega + \frac{1}{jC\omega}} = \boxed{\frac{R/R+r}{1 + j\left(\frac{L\omega}{R+r} - \frac{1}{(R+r)C\omega}\right)} = \underline{H}}$$

4°) $$\underline{H} = \frac{H_0}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \Rightarrow \begin{cases} H_0 = \frac{R}{R+r}. \\ \frac{Q}{\omega_0} = \frac{L}{R+r} \\ Q\omega_0 = \frac{1}{(R+r)C} \end{cases} \Rightarrow Q^2 = \frac{1}{R+r}\cdot\frac{L}{C}$$

$$\Rightarrow \boxed{Q = \frac{1}{R+r}\sqrt{\frac{L}{C}}} \text{ et } \boxed{\omega_0 = \frac{1}{\sqrt{LC}}.}$$

5°) $$H(\omega) = |\underline{H}| = \frac{H_0}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}}$$ max pour $\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = 0 \to \boxed{\omega = \omega_0}$

$H_0 = H(\omega_r) = \max.$

6°) $H(\omega_1) = H(\omega_2) = \frac{H_0}{\sqrt{2}}$; en posant $x = \frac{\omega}{\omega_0}$, on obtient.

$$\frac{H_0}{\sqrt{1+Q^2\left(x-\frac{1}{x}\right)^2}} = \frac{H_0}{\sqrt{2}} \to x - \frac{1}{x} = \pm\frac{1}{Q} \to x^2 \pm \frac{x}{Q} - 1 = 0.$$

$x^2 + \frac{x}{Q} - 1 = 0$ $\qquad$ $x^2 - \frac{x}{Q} - 1 = 0$

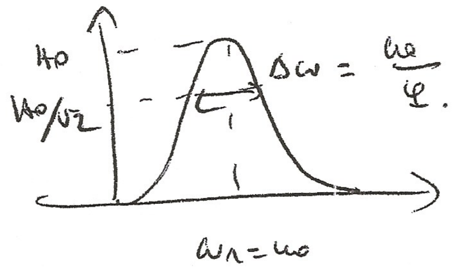
$$\Delta = \frac{1}{Q^2} + 4.$$

$x = -\frac{1}{2Q} \pm \frac{\sqrt{\Delta}}{2}$ $\qquad$ $x = \frac{1}{2Q} \pm \frac{\sqrt{\Delta}}{2}$

En ne gardant que les racines positives $x_1 = -\frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}$

$x_2 = \frac{1}{2Q} + \sqrt{1 + \frac{1}{4Q^2}}$.

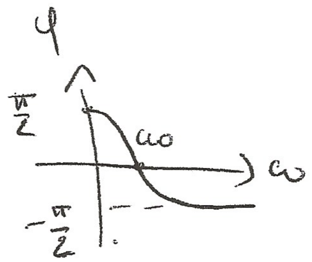
d'où $x_2 - x_1 = \frac{1}{Q} = \frac{\Delta\omega}{\omega_0} \Rightarrow \boxed{Q = \frac{\omega_0}{\Delta\omega}}$

7°) 

8°) $\varphi = -\arctan Q\left(x - \frac{1}{x}\right)$.

$\omega = \omega_r \to x = 1 \to \varphi = 0.$

$\omega = \omega_1$ ou ω_2 } définis par $x - \frac{1}{x} = \pm\frac{1}{Q} \to \varphi = \pm\frac{\pi}{4}$.



Partie 2 Application.

$e(t) = E\cos\omega t \to \underline{E} = E$

$u_s(t) = U_s\cos(\omega t + \varphi) \to \underline{U_s} = U_s.$

BF —m— ⇔ — → $u_s = 0$ }
HF —||— ⇔ — → $u_s = 0$ } Filtre passe bande.

3°) $\underline{H}(j\omega) = \frac{Z_{eq}}{R + Z_{eq}} = \frac{1}{1 + RY_{eq}} = \frac{1}{1 + R\left(jC\omega + \frac{1}{jL\omega}\right)} = \frac{1}{1 + jR\left(C\omega - \frac{1}{L\omega}\right)}$

$$\begin{cases} H_0 = 1 \\ \dfrac{Q}{\omega_0} = RC \\ Q\omega_0 = \dfrac{R}{L} \end{cases} \rightarrow Q^2 = R^2 \cdot \frac{C}{L} \rightarrow \boxed{Q = R\sqrt{\frac{C}{L}}}$$

et $\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$

5°] $\omega_0 = \omega_r \Rightarrow$ pulsation pour laquelle $H(\omega)$ et donc $U_s(\omega)$ est maximal.

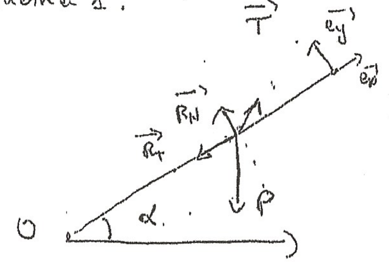
$\omega_0 = \omega_r \Rightarrow \varphi(\omega_0) = 0 \Rightarrow$ d'où $\boxed{\omega_0 = 10^4 \, rad\, s^{-1}}$

$$\left.\begin{aligned} \omega_1 &= 7 \cdot 10^3 = 7000 \, rad\, s^{-1} \\ \omega_2 &= 1{,}6 \cdot 10^4 = 16\,000 \, rad\, s^{-1} \end{aligned}\right\} \Delta\omega = 10000 \, rad\, s^{-1}$$

$$\rightarrow Q = \frac{\omega_0}{\Delta\omega} = \boxed{1 = Q.}$$

Ex3

Question 1.



Question 1.

1ère phase.

$\vec{a} = a\,\vec{e_x}$ avec $a = 0{,}25\ m\,s^{-2}$.

$\vec{OM} = x(t)\,\vec{e_x} \Rightarrow a = \ddot{x}$

$\dot{x}(t) = a.t \quad x(t) = \dfrac{at^2}{2}$ en prenant $\dot{x}(0) = 0$, $x(0) = 0$.

La première phase dure un temps t_1 et $\dot{x}(t_1) = \boxed{v_1 = a t_1}$

Le skieur a parcouru $d_1 = \dfrac{a t_1^2}{2} = \dfrac{a}{2}\dfrac{v_1^2}{a^2} = \boxed{\dfrac{v_1^2}{2a} = d_1}$ et se situe en M_1

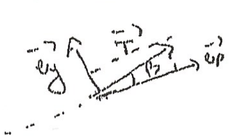
La deuxième phase dure un temps t_2 et $\dot{x}(t) = v_1$ d'où $\boxed{x(t) = v_1 t}$ en prenant comme nouvelle origine (des temps et des abscisses le point M_1).

avec $\boxed{t_2 = \dfrac{d - d_1}{v_1}}$

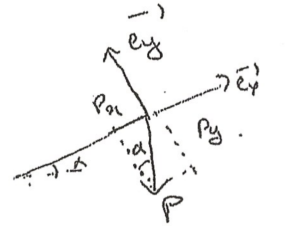
AN $t_1 = \dfrac{v_1}{a_1} = \dfrac{2}{0{,}25} = 8\,s \qquad d_1 = \dfrac{4}{0{,}5} = 8\,m$

$t_2 = \dfrac{492}{2} = 246\,s = 4{,}1\,mn.$

Question 2.



$$\begin{cases} \vec{T} = T\cos\beta\,\vec{e_x} + T\sin\beta\,\vec{e_y} \\ \vec{R_T} = -R_T\,\vec{e_x} \\ \vec{R_N} = R_N\,\vec{e_y} \\ \vec{P} = -mg\sin\alpha\,\vec{e_x} \ldots \cos\alpha\,\vec{e_y} \end{cases}$$



$\vec{P} = -mg\sin\alpha\,\vec{e_x} - mg\cos\alpha\,\vec{e_y}$

En mr $\|\vec{R_T}\| = \mu\|\vec{R_N}\|$.

1ère phase accélérée.

$\vec{P} + \vec{T} + \vec{R_N} + \vec{R_T} = m\vec{a}$.

en proj$/\vec{e_x}$ $\quad -mg\sin\alpha + T\cos\beta - \mu.R_N = ma. \quad (1)$

en proj$/\vec{e_y}$ $\quad -mg\cos\alpha + T\sin\beta + R_N = 0 \quad (2).$

eq (2) $\Rightarrow R_N = mg\cos\alpha - T\sin\beta$

eq (1) $\Rightarrow -mg\sin\alpha + T\cos\beta - \mu(mg\cos\alpha - T\sin\beta) = ma$.

$T[\cos\beta + \mu\sin\beta] = m[a + g\sin\alpha + \mu g\cos\alpha]$.

$$\Rightarrow \boxed{T_1 = \frac{m[a + g(\sin\alpha + \mu\cos\alpha)]}{\cos\beta + \mu\sin\beta}}$$

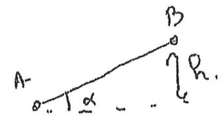
2ème phase uniforme. T_2 se déduit de T_1 en faisant $a = 0$.

$$\boxed{T_2 = \frac{mg(\sin\alpha + \mu\cos\alpha)}{\cos\beta + \mu\sin\beta}}$$

AN $\quad T_1 = 80\,\dfrac{[0{,}25 + 9{,}80(\sin 20^\circ + 0{,}05\cos 20^\circ)]}{\cos 30 + 0{,}05\sin 30}. = \boxed{366\,N = T_1}$

$\boxed{T_2 = 343\,N}$.

Question 4



L = AB

$W_{A\to B}(\vec{P}) = -mgh$ et $\sin\alpha = \dfrac{h}{L}$ $\Rightarrow \dfrac{h}{\sin\alpha}$

$= -mgL\sin\alpha$

$\cdot W_{A\to B}(\vec{R_N}) = 0$

$\cdot W_{A\to B}(\vec{R_T}) = -R_T.L.$

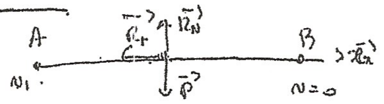
$W_{A\to B}(\vec{T}) = +T.L\cos\beta.$

Lors de la phase uniforme.

$$E_c(B) - E_c(A) = 0 = -mgL\sin\alpha - R_T \cdot L + T \cdot L\cos\beta$$

$$= \Big(\underbrace{-mg\sin\alpha - R_T + T\cos\beta}_{=0 \text{ d'après la projection du PFD suivant } \vec{u_x}}\Big) \cdot L.$$

<u>Question 5.</u>



TEC $E_c(B) - E_c(A) = W_{A\to B}(\vec{R_T})$

$$-\frac{1}{2} m v_1^2 = -d \cdot R_T = -d\mu R_N$$
$$= -d\mu mg.$$

$$\Rightarrow d = \frac{\cancel{m} v_1^2}{2\mu \cancel{m} g} \rightarrow \boxed{d = \frac{v_1^2}{2\mu g}}$$

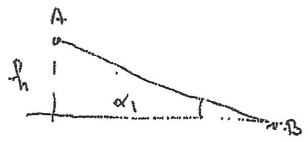
<u>TEM</u> $E_m(B) - E_m(A) = W_{A\to B}(\vec{R_T})$.

<u>AN</u> $d = \frac{4}{2 \cdot 0{,}05 \cdot 9{,}8} = 4\,\text{m}.$

et $x(t) = \frac{1}{2} a t^2 + v_1 t$ et $v(t) = at + v_1$. avec $a = -\frac{R_T}{m} = -\mu g$.

d'où $t_1 = \frac{0 - v_1}{-\mu g} = \frac{v_1}{\mu g} = \frac{2}{0{,}05 \cdot 9{,}8} = 4\,\text{s}.$

<u>Question 6.</u>



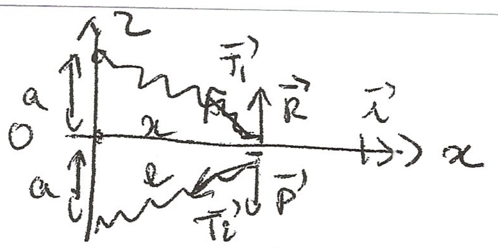
$AB = L$
$\alpha = 45°$.

TEC : $E_c(B) - E_c(A) = \sum W_{A\to B}^{\vec{F}}$

d'où $\frac{1}{2} m v_B^2 = -R_T \cdot L + mgh.$
$$= -\mu mg\cos\alpha L + mgL\sin\alpha.$$

$$\Rightarrow v_B = \left(2\left[g\sin\alpha - \mu g\cos\alpha\right] \cdot L\right)^{1/2} = \left(2gL\left[\sin\alpha - \mu\cos\alpha\right]\right)^{1/2}.$$

<u>AN</u> $v_B = \sqrt{2 \cdot 20 \cdot 9{,}8 \left[\sin 45° - 0{,}05\cos 45°\right]} = \sqrt{\sqrt{2} \cdot 20 \cdot 9{,}8\left[1 - 0{,}05\right]} = 16{,}2\,\text{m}\,\text{s}^{-1}$



1°] a) $\delta W_{\vec{P}} + \delta W_{\vec{R}} + \underbrace{\delta W_{\vec{T_1}} + \delta W_{\vec{T_2}}}_{-dE_p} = dE_c$

$\delta W_{\vec{P}} = 0$, $\delta W_{\vec{R}} = 0$ car $\vec{P}$ et $\vec{R} \perp d\vec{n}$

$$\boxed{\text{avec } E_p = 2 \cdot \frac{1}{2} k (\ell - \ell_0)^2 \text{ et } \ell = \sqrt{a^2 + x^2}}$$

$\Rightarrow d(E_c + E_p) = 0 \to E_c + E_p = \text{cte} \Rightarrow E_m = \frac{1}{2} m v^2 + \frac{2}{2} \cdot k (\ell - \ell_0)^2$.

or $\vec{OM} = x\vec{i} \to \vec{v} = \dot{x}\vec{i}$ d'où $E_m = \frac{1}{2} m \dot{x}^2 + k\left(\sqrt{a^2+x^2} - \ell_0\right)^2 = \text{ste}$

b) $\frac{dE_m}{dt} = 0 \Rightarrow \frac{1}{2} m \cdot 2\dot{x}\ddot{x} + \frac{2k x \dot{x}}{\sqrt{a^2+x^2}}\left(\sqrt{a^2+x^2} - \ell_0\right) = 0$.

d'où $$\boxed{\ddot{x} + \frac{2k}{m} x \left(1 - \frac{\ell_0}{\sqrt{a^2+x^2}}\right) = 0}$$ éq. diff non linéaire.

2°] $m\ddot{x} = T_{x1} + T_{x2} = -\frac{dE_p}{dx}$. d'où les positions d'éq vérifient

$\frac{dE_p}{dx} = 0$ or $\frac{dE_p}{dx} = \frac{2kx}{\sqrt{a^2+x^2}}\left[\sqrt{a^2+x_0^2} - \ell_0\right] = 0$

$$\Rightarrow \boxed{\begin{cases} x_{e1} = 0 \\ x_{e_{2,3}} = \pm\sqrt{\ell_0^2 - a^2} \quad \text{si } \ell_0 > a. \end{cases}}$$ et $$\boxed{\begin{matrix} x_{e1} = 0 \\ \text{si } \ell_0 < a. \end{matrix}}$$

$\frac{d^2E_p}{dx^2} = \frac{d}{dx}\left[2kx\left(1 - \frac{\ell_0}{\sqrt{a^2+x^2}}\right)\right] = 2k\left[1 - \frac{\ell_0}{\sqrt{a^2+x^2}}\right] + \frac{2kx\ell_0 x}{(a^2+x^2)^{3/2}}$

d'où $\frac{d^2E_p}{dx^2}(x=0) = 2k\left(1 - \frac{\ell_0}{a}\right)$ >0 si $\ell_0 < a$. $\Rightarrow$ $x_{e1} = 0$ stable

<0 si $\ell_0 > a$. $x_{e1} = 0$ instable.

$$\frac{d^2E_p}{dx^2}\left(\begin{matrix} x_{e_2} \\ \text{a} \\ x_{e_3} \end{matrix}\right) = +\frac{2k\ell_0\left(\ell_0^2-a^2\right)}{\left(\ell_0^2\right)^{3/2}} = +2k\left(1-\frac{a^2}{\ell_0^2}\right) > 0 \quad \text{stable.}$$

3°] $m\ddot{x} = -\dfrac{dE_p}{dx}$

Autour de x_e

$$\frac{dE_p}{dx}(x) = \underbrace{\frac{dE_p}{dx}(x_e)}_{=0} + (x-x_e)\frac{d^2E_p}{dx^2}(x_e).$$

d'où $\ddot{x} + \dfrac{\frac{d^2E_p}{dx^2}(x_e)}{m}(x-x_e) = 0$

où $\ddot{(x-x_e)} + \omega_0^2\,(x-x_e) = 0$ avec

$\boxed{\omega^2 = \dfrac{2k}{m}\left(1-\dfrac{\ell_0}{a}\right)}$ autour de $x_{e_1} = 0$ $(a > \ell_0)$

$\boxed{\omega'^2 = \dfrac{2k}{m}\left(1-\dfrac{a^2}{\ell_0^2}\right)}$ autour de x_{e_2} ou x_{e_3}.

$(a < \ell_0)$

et $\boxed{\omega_0^2 = \dfrac{2k}{m}.}$