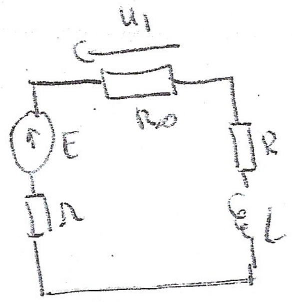
Partie B Arrêt du courant.

1°)

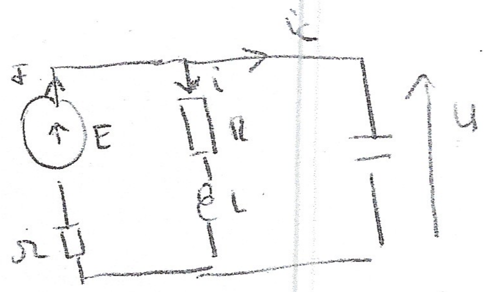


K fermé

$i(0^-) = \frac{E}{R+r}$.

K ouvert, l'interrupteur est assimilable à une résistance très grande, notée R_∞ et $i(0^+) = \frac{E}{R+r}$ par continuité $\Rightarrow u_1 = R_\infty i(0^+) = R_\infty \cdot \frac{E}{R+r} \nearrow \infty \rightarrow$ l'air devient conducteur

2°]

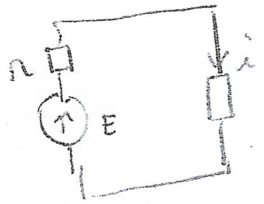


$$\begin{cases} E = rI + u & (1) \\ I = i + i_C & (2) \\ u = Ri + L\frac{di}{dt} & (3) \\ i_C = C\frac{du}{dt} & (4) \end{cases}$$

3°]

à $t = 0^-$ $i(0^-) = 0 \Rightarrow i(0^+) = 0$
$u(0^-) = 0 \rightarrow u(0^+) = 0$ $\Big\}$ et (3) donne $\frac{di}{dt}(0^+) = 0$

4°] à l'∞ $u_L = L\frac{di}{dt} = 0$ et $i_C = 0$.



$i(\infty) = \frac{E}{r+R}$ et $u(\infty) = \frac{R}{r+R}$ on retrouve les résultats de la partie A.

5°] (1) + (2) donnent $E = r(i + i_C) + u = ri + rC\frac{du}{dt} + Ri + L\frac{di}{dt}$.

d'où $E = (r+R)\,i + rC\left[R\frac{di}{dt} + L\frac{d^2i}{dt^2}\right] + L\frac{di}{dt}$

$$rLC\frac{d^2i}{dt^2} + (RC\cdot r + L)\frac{di}{dt} + (r+R)\,i = E.$$

$$LC\frac{d^2i}{dt^2} + \left(RC + \frac{L}{r}\right)\frac{di}{dt} + \left(\frac{r+R}{r}\right) i = \frac{E}{r}$$

d'où

$$\boxed{\frac{d^2i}{dt^2} + \frac{\left(RC + \frac{L}{r}\right)}{LC}\frac{di}{dt} + \left(\frac{\frac{R+r}{r}}{LC}\right) i = \frac{E}{r\cdot LC}.}$$

On obtient $\begin{cases} \omega_0^2 = \dfrac{1}{LC} \\ 2\lambda\omega_0 = \dfrac{\left(RC + \frac{L}{r}\right)}{LC} \end{cases}$

Resolution

e^{rt} est solution $\Rightarrow$ $r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$\rightarrow$ $\Delta = 4\lambda^2\omega_0^2 - 4\omega_0^2 = 4\omega_0^2(\lambda^2 - 1)$

$r = -\lambda\omega_0 \pm j\omega_0\sqrt{1-\lambda^2}$ $\Rightarrow$

$i(t) = A e^{-\lambda\omega_0 t} \cos(\omega_0\sqrt{1-\lambda^2}\,t + \varphi) + \dfrac{E}{R+r}$.

à $t = 0^+$ $i(0^+) = 0$ $\Rightarrow$ $A\cos\varphi + \dfrac{E}{R+r} = 0$

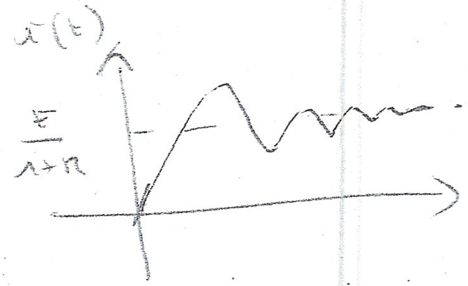
$\dfrac{di}{dt} = 0 \Rightarrow -\lambda\omega_0\cos\varphi - \omega_0\sqrt{1-\lambda^2}\sin\varphi = 0$

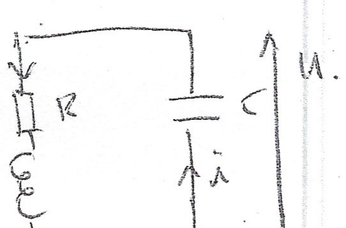
d'où $\begin{cases} A\cos\varphi + \dfrac{E}{r+R} = 0 \\ \lambda\cos\varphi + \sqrt{1-\lambda^2}\sin\varphi = 0 \end{cases}$ $\Rightarrow$ $\operatorname{tg}\varphi = \dfrac{\sin\varphi}{\cos\varphi} = \cancel{\dfrac{-\sqrt{1-\lambda^2}}{\lambda}}\ \dfrac{-\lambda}{\sqrt{1-\lambda^2}}$

or $A = -\dfrac{\frac{E}{r+R}}{\cos\varphi}$ avec $\dfrac{1}{\cos^2\varphi} = 1 + \operatorname{tg}^2\varphi = 1 + \dfrac{\lambda^2}{1-\lambda^2} = \dfrac{1}{1-\lambda^2}$.

$\Rightarrow$ $\cos\varphi = \sqrt{1-\lambda^2}$

d'où $\boxed{\begin{cases} i(t) = \dfrac{E}{R+r}\left(1 - \dfrac{e^{-\lambda\omega_0 t}}{\sqrt{1-\lambda^2}}\cos(\omega_0\sqrt{1-\lambda^2}\,t + \varphi)\right) \\ \text{avec } \varphi = -\operatorname{arctg}\dfrac{\lambda}{\sqrt{1-\lambda^2}}. \end{cases}}$





$i(0^-) = \dfrac{E}{r+R} \Rightarrow \boxed{i(0^+) = \dfrac{E}{r+R}.}$

$u(0^+) \cancel{=i}$ $\boxed{u(0^-) = \dfrac{R}{R+r}\cdot E = u(0^+)}$

$u(0^+) = R\,i(0^+) + L\dfrac{di(0^+)}{dt} \Rightarrow \dfrac{di(0^+)}{dt} = \dfrac{1}{L}\left[\dfrac{R}{r+R}E - \dfrac{R\,E}{R+r}\right]$

(5).

$\boxed{0 = \dfrac{di(0^+)}{\ }}$

+ l'étincelle est évitée car le courant dans la bobine "s'écoule" dans la branche du condensateur.

b]
$$\begin{cases} u = Ri + L\dfrac{di}{dt} \\ i = -C\dfrac{du}{dt} \end{cases} \rightarrow \text{En } \cancel{Ri +} \text{ dérivant}$$

$$-\frac{i}{C} = R\frac{di}{dt} + L\frac{d^2i}{dt^2}$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0.}$$

$$2\lambda\omega_0 = \frac{R}{L}$$

$$\omega_0^2 = \frac{1}{LC}$$

On suppose le régime transitoire critique d'où $\lambda = 1$

et $\boxed{i(t) = (A + Bt)e^{-\omega_0 t}}$

à $t = 0 \Rightarrow A = \dfrac{E}{r+R}$.

$i(0^+) = \dfrac{E}{r+R}$.

$$\frac{di}{dt} = -\omega_0(A + Bt)e^{-\omega_0 t} + Be^{-\omega_0 t}$$

$$\frac{di}{dt}(0^+) = -\omega_0 A + B = 0 \rightarrow B = \omega_0 A.$$

$$\boxed{i(t) = \frac{E}{r+R}\left(1 + \omega_0 t\right)e^{-\omega_0 t}.}$$