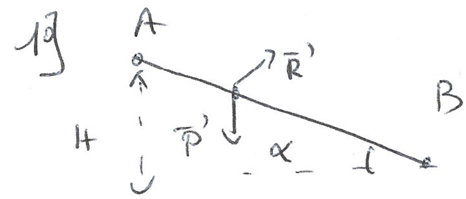
Correction DS4 23-24.

Ex 1 Saut à ski.

1°]



$L = AB = 40\,m$ $\quad Ec(B) - Ec(A) = W_{A\to B}(\vec{P}) + \underbrace{W_{A\to B}(\vec{R})}_{=0 \text{ car } \vec{R} \perp d\vec{M}}$

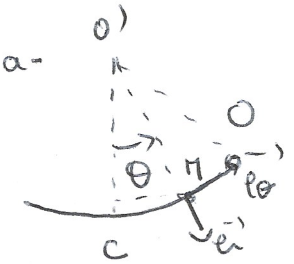
$\alpha = 30°$

et $W_{\vec{P}} = +mgH = mgL\sin\alpha$.

$V_A = 0 \rightarrow Ec(B) = \frac{1}{2} m V_B^2 = mgL\sin\alpha \rightarrow \boxed{V_B = \sqrt{2gL\sin\alpha}}$

<u>AN</u> $g = 10\,m\,s^{-2} \rightarrow \boxed{V_B = 20\,m\,s^{-1}}$

2°] a-



$\vec{O'M} = b\,\vec{e_r}$.

$dEc = \delta W_{\vec{P}} + \underbrace{\delta W_{\vec{R}}}_{=0} = -dEp \rightarrow Ec + Ep = cste \rightarrow$ l'énergie mécanique se conserve et $Ep(M) = -mgb\cos\theta + cste$.

$Ep(O) = -mgb\cos\beta + cste = 0 \rightarrow cste = mgb\cos\beta \Rightarrow \boxed{Ep(M) = mg(\cos\beta - \cos\theta)\,b}$

b- en B $\theta = -\alpha \rightarrow \boxed{Ep(B) = mgb(\cos\beta - \cos\alpha)}$

$\boxed{Ep(C) = mgb(\cos\beta - 1)}$

3°] $Em = cste \rightarrow \frac{1}{2} m V_B^2 + Ep(B) = \frac{1}{2} m V(C)^2 + Ep(C) \rightarrow \frac{1}{2} m V(C)^2 = mgL\sin\alpha + Ep(B) - Ep(C)$.

or $Ep(B) - Ep(C) = mgb(\cos\beta - \cos\alpha) - mgb(\cos\beta - 1) = mgb(1 - \cos\alpha)$

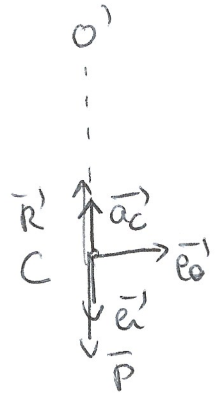
$\Rightarrow \frac{1}{2} \cancel{m} V_C^2 = \cancel{m} gL\sin\alpha + \cancel{m} gb(1 - \cos\alpha) \Rightarrow \boxed{V_C = \sqrt{2g\left[L\sin\alpha + b(1 - \cos\alpha)\right]}}$.

<u>AN</u> $g = 10\,m\,s^{-2}$ $\quad \alpha = 30°$. $\quad \boxed{V_C = 21{,}4\,m\,s^{-1}}$
$L = 40\,m$.
$b = 20\,m$.

$\vec{OM} = b\vec{e_r}$ $\quad \vec{v} = b\dot{\theta}\vec{e_\theta}$ et $\vec{a} = b\ddot{\theta}\vec{e_\theta} - b\dot{\theta}^2\vec{e_r}$. or $\vec{P} + \vec{R}$ vertical donc suivant $\vec{e_r}$ au point C $\Rightarrow \ddot{\theta} = 0$ en C et $\vec{a_C} = -b\dot{\theta}^2\vec{e_r} = -\frac{V_C^2}{b}\vec{e_r} = -2g\left[\frac{L}{b}\sin\alpha + (1 - \cos\alpha)\right]\vec{e_r}$

$$\boxed{\overrightarrow{a_C} = \ominus\, 2g\left[\frac{L}{b}\sin\alpha + (1-\cos\alpha)\right]\overrightarrow{e_r}}$$

4°]



au point C :

$$\vec{P} + \vec{R} = m\overrightarrow{a_C} \qquad \vec{R} = -R\,\overrightarrow{e_r} \qquad \vec{P} = mg\,\overrightarrow{e_r}$$

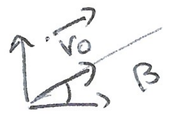
en proj/$\overrightarrow{e_r}$: $+mg - R = -m\dfrac{v_C^2}{b}$

$$\Rightarrow R = mg + m\frac{v_C^2}{b}$$

$$= mg + 2mg\left(\frac{L}{b}\sin\alpha + (1-\cos\alpha)\right).$$

$$\boxed{R(C) = 3mg + 2mg\left(\frac{L}{b}\sin\alpha - \cos\alpha\right).}$$

5°]



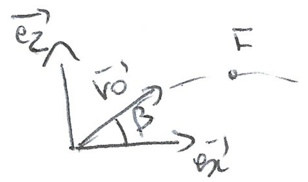
$$\overrightarrow{V_0} = V_0\cos\beta\,\overrightarrow{e_x} + V_0\sin\beta\,\overrightarrow{e_z}.$$

Calcul de v_0 : $\quad E_m(B) = E_m(0) \quad$ avec $E_p(0) = 0$.

$$\frac{1}{2}m v_0^2 = \frac{1}{2}m v_B^2 + mgb(\cos\beta - \cos\alpha) \Rightarrow \cancel{m}gL\sin\alpha + \cancel{m}gb(\cos\beta - \cos\alpha).$$

$$\boxed{v_0 = \left(2g\left[L\sin\alpha + b(\cos\beta - \cos\alpha)\right]\right)^{1/2}.}$$

6°]



$$m\vec{a} = m\vec{g} \to \quad m\ddot{x} = 0 \to \dot{x} = v_0\cos\beta \to x(t) = v_0\cos\beta\, t$$

$$m\ddot{z} = -mg \to \dot{z} = -gt + v_0\sin\beta \to z(t) = -g\frac{t^2}{2} + v_0\sin\beta\, t.$$

en F : $\dot{z} = 0 \to t_F = \dfrac{v_0\sin\beta}{g}$.

$$z_F = -\frac{g}{2}\left(\frac{v_0\sin\beta}{g}\right)^2 + \frac{v_0^2\sin^2\beta}{g} \to z_F = \frac{v_0^2\sin^2\beta}{2g}.$$

$$\Rightarrow \boxed{z_F = \left[L\sin\alpha + b(\cos\beta - \cos\alpha)\right]\sin^2\beta} \quad \underline{\text{AN}} \quad \boxed{z_F = 6{,}3\text{ m}.}$$

exo 2.

Q1 $E_c(A) = \frac{1}{2} m v_A^2$ Q2 $E_c(B) = \frac{1}{2} m v_B^2$ → $\Delta E_c = E_c(B) - E_c(A) = \frac{1}{2} m (v_B^2 - v_A^2)$.

Q3 $\Delta E_c = -2{,}2 \cdot 10^8\ J$.

Q4 $E_c(B) - E_c(A) = \sum W_{A \to B}(\vec{F}/m)$.

Q5 $W_{AB}(\vec{P}) = -mg\Delta h_{AB} > 0$. $W_{AB}(\vec{P}) = mg(z_A - z_B)$ $z_A = 10{,}6$ km
$z_B = 7{,}5$ km.

car $\vec{P}$ et $\vec{dn}$ sont tels
que $\vec{P} \cdot \vec{dn} > 0$.

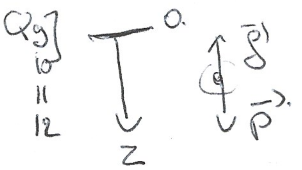
AN $W_{AB}(\vec{P}) = 34\,\mathrm{nJ}$.

Q6 Travail moteur car $W_{AB}(\vec{P}) > 0$.

Q7 $\Delta E_c = E_c(B) - E_c(A) = W_{AB}(\vec{P}) + W_{AB}(\vec{f})$ ⇒ $\boxed{W_{AB}(\vec{f}) \simeq -\cancel{2{,}8} \cdot 10^8\ J}$.

Q8 si $\vec{f}$ cte $W_{AB}(\vec{f}) = -f \cdot (z_A - z_B)$ ⇒ $f = \frac{W_{AB}(\vec{f})}{z_B - z_A} = \boxed{10^5\ N = f}$.

Q9



Presence de frottement fluide → ce n'est pas une chute libre.

$\vec{OG} = z\vec{e_z}$ $\vec{v} = \dot{z}\vec{e_z} = v\vec{e_z}$, $\vec{a} = \dot{v}\vec{e_z}$ $\vec{P} = mg\vec{e_z}$ $\vec{f} = -hv\vec{e_z}$

2ème loi de Newton $m\vec{a} = \vec{P} + \vec{f}$ projetée suivant $\vec{e_z}$

$mg - hv = m\dot{v}$ → $\dot{v} + \frac{h}{m} v = g$ $\boxed{B = g}$ $\boxed{A = \frac{h}{m} = \frac{1}{\tau}}$

$t > 10\tau$ $\boxed{v \to v_{lg} = v_{lim} = \frac{mg}{h}}$

Q13 $\lambda_0 = cT = \frac{c}{f}$ → $\boxed{\lambda_0 = 1\ \text{cm}}$

Q14. $H = \frac{c\Delta t}{2}$ (distance = 2H sur un aller retour). H = 7,5 km (activation du radar).

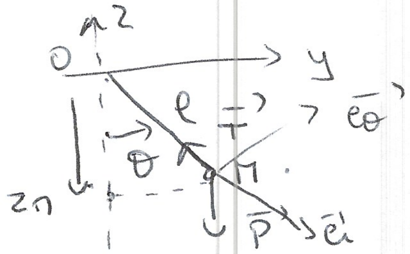
$\Delta t = 50\,\mu s$.

Problème.

1] La période T peut dépendre de la longueur ℓ, de g et de m.

$[T] = T$ $\quad [\ell] = L$ $\quad [g] = L.T^{-2}$ $\quad [m] = M \Rightarrow$ ne peut pas intervenir ds T.

$\left[\frac{\ell}{g}\right] = T^2 \Rightarrow \boxed{T = A\sqrt{\frac{\ell}{g}}}$



2°] $E_m = \frac{1}{2}mv^2 + E_p(\vec{P})$.

$dE_c = \delta W_{\vec{P}} + \underbrace{\delta W_{\vec{T}}}_{0 \text{ car } \vec{T} \perp \vec{dM}}$

$dE_c = -dE_p \rightarrow \boxed{E_c + E_p = E_m = cste}$

* $\vec{OM} = \ell \vec{e_r}$ $\quad \vec{v} = \ell\dot{\theta}\vec{e_\theta} \Rightarrow E_c = \frac{1}{2}m\ell^2\dot{\theta}^2$.

* axe z ascendant $\Rightarrow E_p(\vec{P}) = +mgz + cste$, avec $z_M < 0$ et $z_M = -\ell\cos\theta$.

$\Rightarrow E_p(M) = -mg\ell\cos\theta + cste$. $\Rightarrow \boxed{E_m = \frac{1}{2}m\ell^2\dot{\theta}^2 - mg\ell\cos\theta + cste}$

3°] $\frac{dE_m}{dt} = 0 \quad \cancel{\frac{1}{2}}\cancel{m}\ell^2 \cancel{2}\ddot{\theta}\cdot\cancel{\dot{\theta}} + \cancel{m}g\cancel{\ell}\cancel{\dot{\theta}}\sin\theta = 0 \Rightarrow \boxed{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}$

4°] 2ème loi de Newton : $\vec{T} + m\vec{g} = m\vec{a}$, avec $\vec{T} = -T\vec{e_r}$

$\vec{P} = mg\cos\theta\,\vec{e_r} - mg\sin\theta\,\vec{e_\theta}$ $\quad \vec{a} = -\ell\dot{\theta}^2\vec{e_r} + \ell\ddot{\theta}\vec{e_\theta}$.

PFD $/\vec{e_\theta}$ $\quad m\ell\ddot{\theta} = -mg\sin\theta \Rightarrow \boxed{\ddot{\theta} + \frac{g}{\ell}\sin\theta = 0}$

5°] Petites oscillations : $\sin\theta \simeq \theta \rightarrow \ddot{\theta} + \frac{g}{\ell}\theta = 0$

$\rightarrow \omega_0 = \frac{g}{\ell}$ et $T_0 = \frac{2\pi}{\omega_0} = \boxed{2\pi\sqrt{\frac{\ell}{g}} = T_0}$ $\quad \theta(t) = A\cos(\omega_0 t + \varphi)$

à $t=0$ $\theta = \theta_0 \rightarrow \theta_0 = A\cos\varphi \quad \rightarrow A = \theta_0 \quad \boxed{\theta(t) = \theta_0\cos\omega_0 t}$

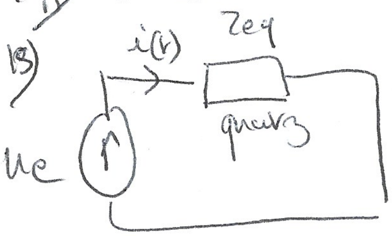
$\dot{\theta} = 0 \quad 0 = -A\omega_0\sin\varphi \rightarrow \varphi = 0$

(4)

6°] $\boxed{\ell = g\,\frac{T_0^2}{4\pi^2} = 0{,}99\,m}$.

II Etude d'un circuit à quartz.

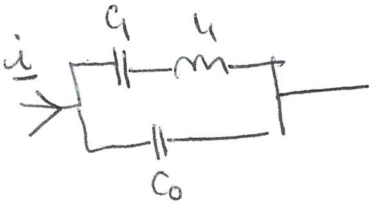
15)



loi des mailles $u_e - Z_{eq}\cdot \underline{i} = 0$

$$\underline{i} = \frac{u_e}{Z_{eq}}.$$

16)



$$Y_{eq} = jC_0\omega + \frac{1}{jL_1\omega + \frac{1}{jC_1\omega}} = jC_0\omega + \frac{jC_1\omega}{1-L_1C_1\omega^2}.$$

$$= \frac{jC_0\omega\,(1-L_1C_1\omega^2) + jC_1\omega}{1-L_1C_1\omega^2} = \frac{j(C_0+C_1)\omega - jC_0L_1C_1\omega^3}{1-L_1C_1\omega^2}.$$

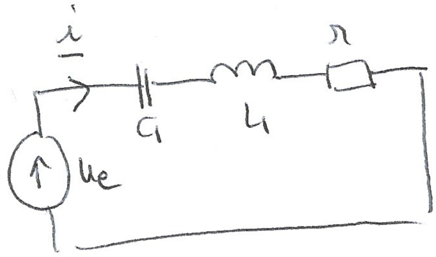
$$Y_{eq} = j(C_0+C_1)\omega\,\frac{\left(1-\frac{C_0}{C_0+C_1}L_1C_1\omega^2\right)}{1-L_1C_1\omega^2} = \boxed{jC_{eq}\,\omega\,\frac{\left(1-\frac{\omega^2}{\omega_2^2}\right)}{1-\frac{\omega^2}{\omega_1^2}}} = Y_{eq}.$$

donc $\boxed{\omega_1 = \frac{1}{\sqrt{L_1C_1}}}$ $\boxed{\omega_2 = \frac{1}{\sqrt{\frac{C_0C_1}{C_0+C_1}L_1}}}$ et $\boxed{C_{eq} = C_0+C_1}$.

17] $\frac{1}{Z_{eq}} \to \infty$ lorsque $1-\frac{\omega^2}{\omega_1^2} = 0 \to \boxed{\omega_r = \omega_1 = \frac{1}{\sqrt{L_1C_1}}}$ $\boxed{f_1 = \frac{1}{2\pi\sqrt{L_1C_1}}}$

et l'amplitude de i devient très grande.

18]



$$\underline{i} = \frac{u_e}{r + jL_1\omega + \frac{1}{jC_1\omega}} = \frac{u_e/r}{1+j\left(\frac{L_1}{r}\omega - \frac{1}{rC_1\omega}\right)}.$$

$$= \frac{u_e/r}{1+jQ\left(\frac{\omega}{\omega_1} - \frac{\omega_1}{\omega}\right)}$$

$$\Rightarrow \begin{cases} \frac{Q}{\omega_1} = \frac{L_1}{r} \\ Q\cdot\omega_1 = \frac{1}{rC_1} \end{cases} \rightarrow Q^2 = \frac{L_1}{r^2C_1} \rightarrow \boxed{Q = \frac{1}{r}\sqrt{\frac{L_1}{C_1}}}$$

et $\boxed{\omega_1 = \frac{1}{\sqrt{L_1C_1}}}$.

$$\left.\begin{array}{l}[\omega_1] = T^{-1} \\ \text{et } [\mathcal{Q}] = 1\end{array}\right\}$$ d'après la factor canonique.

__Vérification__. a) Sachant que $[L\omega] = \left[\frac{1}{C\omega}\right] = [R]$.

$$\Downarrow$$

$\omega_1 = \frac{1}{\sqrt{L_1 C_1}}$ donc $[L\cdot C][\omega]^2 = 1.$ $\rightarrow$ donc $[LC] = T^2$

d'où $\sqrt{L_1C_1} = T$ $\Rightarrow$ $\frac{1}{\sqrt{L_1C_1}} = T^{-1}$.

$[\omega_1] = T^{-1}$ vérifié

b) $[L\omega] = \frac{1}{[C\omega]} = [R] \Rightarrow$ $[L\omega]\cdot\frac{1}{[C\omega]} = [R]^2$ $\Rightarrow$ $\sqrt{\left[\frac{L}{C}\right]} = [R]$.

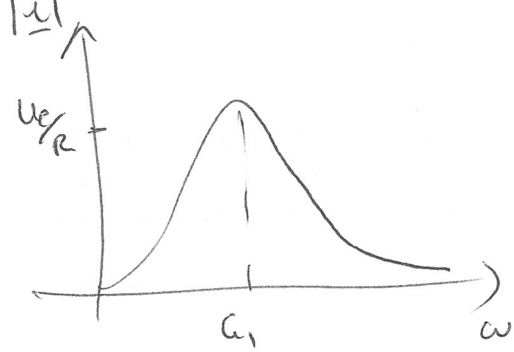
d'où $[\mathcal{Q}] = \frac{1}{[R]}\sqrt{\left[\frac{L_1}{C_1}\right]} = 1$. $[\mathcal{Q}] = 1$ vérifié.

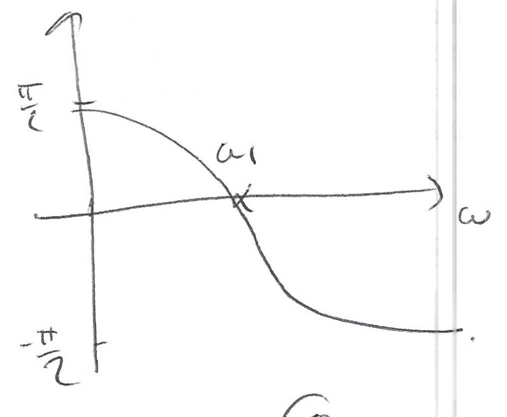
$|\underline{i}| = \frac{U_e/R}{\sqrt{1+\mathcal{Q}^2\left(\frac{\omega}{\omega_1}-\frac{\omega_1}{\omega}\right)}}$ $|\underline{i}| \rightarrow 0$ pour $\omega \rightarrow 0$ et $\omega \rightarrow \infty$. $|\underline{i}| = U_e/R$ pour $\omega = \omega_1$.

$\varphi = - \arctan \mathcal{Q}\left(\frac{\omega}{\omega_1}-\frac{\omega_1}{\omega}\right)$ $\Rightarrow$ $\varphi = -\arctan \cancel{\infty} = \frac{\pi}{2}$ pour $\omega \rightarrow 0$

$\varphi = -\arctan\infty = -\frac{\pi}{2}$ pour $\omega \rightarrow \infty$.

$\varphi = 0$ pour $\omega = \omega_1$





19. $\underline{u_S} = R_0\, \underline{i}$

a- $\Rightarrow \underline{u_S} = \dfrac{\overbrace{\frac{R}{r} \cdot u_0}^{U_{max}}}{1 + jQ\left(\frac{\omega}{\omega_1} - \frac{\omega_1}{\omega}\right)}$.

On pose $x = \dfrac{\omega}{\omega_1}$.

$|\underline{u_S}(x_1)| = |\underline{u_S}(x_2)| = \dfrac{U_{Smax}}{\sqrt{2}} \Rightarrow 1 + Q^2\left(x - \frac{1}{x}\right)^2 = 2 \rightarrow \left(x - \frac{1}{x}\right) = \pm \dfrac{1}{Q}$.

$x^2 - \dfrac{x}{Q} - 1 = 0$ ou $x^2 + \dfrac{x}{Q} - 1 = 0$

$\Delta = \dfrac{1}{Q^2} + 4$

$x_2 = \dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}$ $\qquad x_1 = -\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}$

$\Rightarrow \boxed{f_1 = f_r\left(-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right). \quad f_2 = f_r\left(\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right).}$

Rq $f_r = \dfrac{\omega_r}{2\pi} = \dfrac{\omega_1}{2\pi}$ ← question 18.

b-. A la resonance $u_S = \dfrac{R}{r} \cdot u_0$ avec $R = 47\,k\Omega$, $u_0 = 0{,}20\,V$. On a $u_S = 4{,}3\,V$

$\rightarrow \boxed{r = 2{,}2\,k\Omega.}$

20. Mesure de Δf $\quad u_S(f_1) = u_S(f_2) = \dfrac{4{,}3}{\sqrt{2}} = 3\,V$ $\quad \boxed{\Delta f = 1{,}5\,Hz}$.

$f_r = 32768\,Hz$

$Q = \dfrac{32768}{1{,}5} = 20000$.

21] $\boxed{L_1 = \dfrac{rQ}{\omega_1}}$ $\quad \boxed{C_1 = \dfrac{1}{\omega_1 r Q}}$ $\quad$ AN $L_1 = 200\,H$.

Rq La valeur de L_1 n'est pas la valeur usuelle de composant utilisé en electronique mais represente ici un outil servant à la modelisation du quartz.