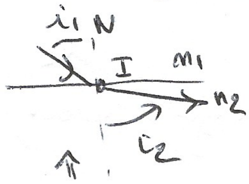
Correction DS2 20/10/23

Ex1 Optique.

1] lois de la réfraction

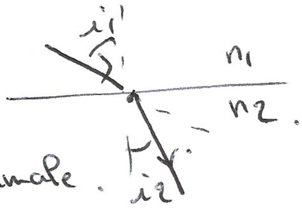
a] $n_{milieu} = \frac{c_{vide}}{c_{milieu}}$ avec $c_{milieu} < c_{vide}$ $\Rightarrow n > 1$.

b] 

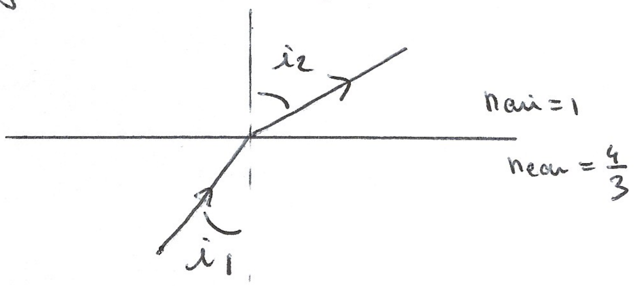
$\begin{cases} \text{Rayon réfracté} \in \text{plan d'incidence} = (\text{rayon incident}, IN). \\ n_1 \sin i_1 = n_2 \sin i_2 \quad \Rightarrow \quad n_1 > n_2, \; i_1 < i_2. \end{cases}$

$i_2 > i_1 \Rightarrow n_2 < n_1$.
rayon réfracté s'écarte de la normale.

si $i_2 < i_1$ $n_2 > n_1$
le rayon réfracté
se rapproche de la normale.



c] La réflexion totale se fait lorsque les rayons incidents se propagent ds l'eau pour se réfracter ds l'air.

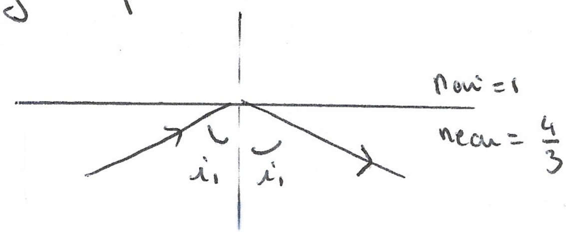


$n_{eau} \sin i_1 = \sin i_2$.

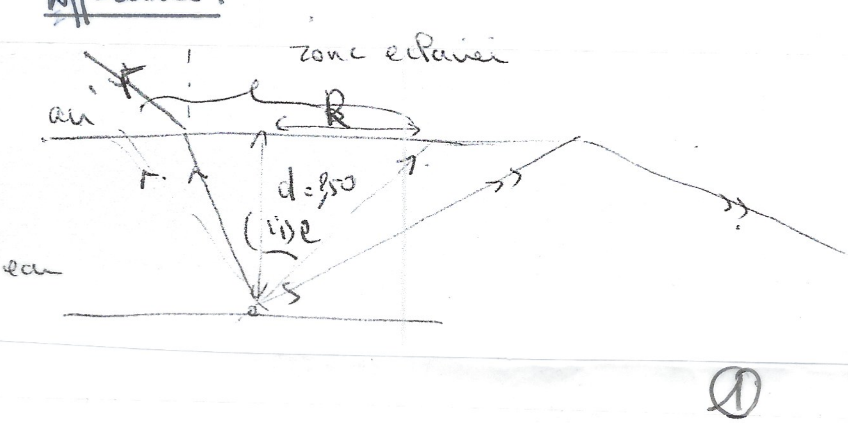
i_2 défini si $n_{eau} \sin i_1 < 1$

$i_1 < \arcsin \frac{1}{n_{eau}} = 48{,}5°$.

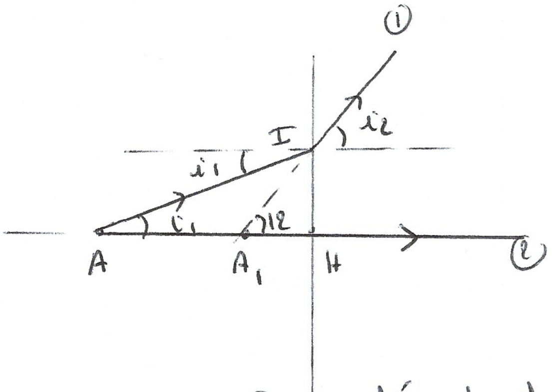
Il y a réflexion totale si $i_1 > 48{,}5°$ $\Rightarrow$



Application.



$\tan i_e = \frac{R}{d} \rightarrow 2R = 5{,}68\,m$.



$n_1 > n_2$
$i_1 < i_2$

- Si A' existe, A' est l'intersection des rayons ① et ②
- On calcule la position de A_1 et on examine si elle dépend du rayon AI (par l'intermédiaire de i_1) auquel cas A_1 n'est pas unique et le dioptre n'est pas stigmatique rigoureux.

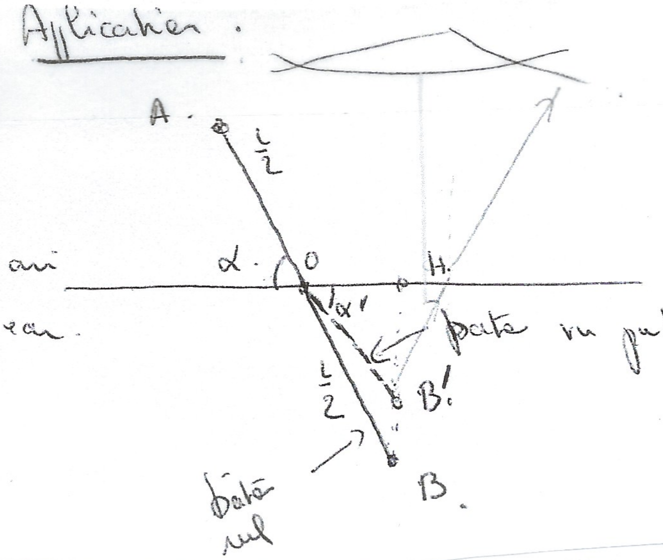
$$tg\, i_1 = \frac{HI}{AH} \text{ et } tg\, i_2 = \frac{HI}{A_1H} \to tg\, i_1 . AH = A_1H\, tg\, i_2 \to \frac{\sin i_1}{\cos i_1} AH = \frac{\sin i_2}{\cos i_2} . A_1H.$$

$$A_1H = \frac{\cos i_2}{\cos i_1} \cdot \frac{\sin i_1}{\sin i_2} AH = \frac{\cos i_2}{\cos i_1} \cdot \frac{n_2}{n_1} AH \implies A_1 \text{ n'est pas unique.}$$

Stigmatisme approché $i_1 \to 0 \implies i_2 \to 0$ et $\cos i_1 \simeq 1$ $\cos i_2 \simeq 1$

$$\implies \boxed{\frac{\overline{HA'}}{n_2} = \frac{\overline{HA}}{n_1} \quad A \xrightarrow{n_1/n_2} A'}$$

Application :



l'observateur voit l'image de B (et de ts les points compris entre O et B) par le dioptre eau/air = B'

$$B \xrightarrow{\text{eau/air}} B'$$

$$\frac{\overline{HB'}}{1} = \frac{\overline{HB}}{\frac{4}{3}} \implies \overline{HB'} = \frac{3}{4} HB$$

$$tg\, \alpha' = \frac{HB'}{OH}$$

$$tg\, \alpha = \frac{HB}{OH} \implies \boxed{tg\, \alpha' = \frac{3}{4} tg\, \alpha}$$

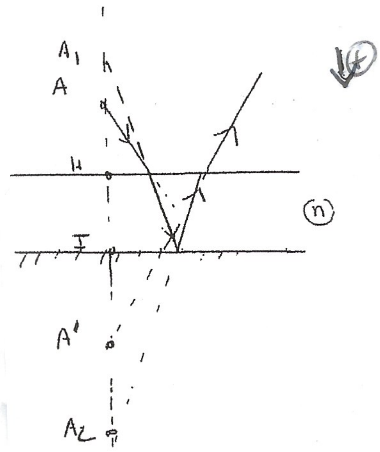
3- Miroir

1) $A \xrightarrow{1/n} A_1 \xrightarrow[\text{Plan}]{\text{Miroir}} A_2 \xrightarrow{n/1} A'$

$\ast$ $\frac{\overline{HA_1}}{n} = \overline{HA}$ avec $\overline{HA} = -h$ $\Rightarrow$ $\boxed{\overline{HA_1} = -n.h.}$

AN $\overline{HA_1} = -\frac{3}{2} \cdot 0{,}8 = \boxed{-1{,}2\text{ cm} = HA_1}$

2)



$\ast\ast$ $\overline{JA_2} = -\overline{JA_1}$ avec $\overline{JA_1} = \overline{JH} + \overline{HA_1} = -e - nh.$

$\boxed{\overline{JA_2} = e + nh.}$ AN $JA_2 = 0{,}\cancel{3} + 1{,}2 =$
$= 1{,}5\text{cm}.$

$\ast\ast\ast$ $\frac{\overline{HA_2}}{n} = \overline{HA'}$ $\Rightarrow$ $\overline{HA'} = \frac{\overline{HJ} + \overline{JA_2}}{n} = \frac{e + e + nh}{n.}$

$\boxed{\overline{HA'} = \frac{2e}{n} + h.}$ AN $\overline{HA'} = \frac{0{,}\cancel{3}\cdot 4}{3} + 0{,}8 =$
$= 1{,}2\text{ cm}$

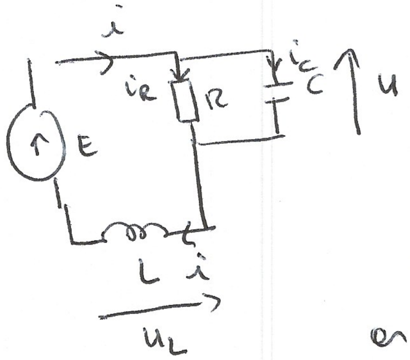
c Le système fonctionne comme un MP unique placé en H_0 avec $\overline{A'H_0} = -\overline{AH_0}$ ou $\overline{AA'} = 2\overline{AH_0}$

or $\overline{AA'} = \overline{AH} + \overline{HA'} = h + \frac{2e}{n} + h = 2h + \frac{2e}{n} = 2\overline{AH_0}$ $\Rightarrow$

$\overline{AH_0} = h + \frac{e}{n}.$ et $\overline{HH_0} = \overline{HA} + \overline{AH_0} = -h + h + \frac{e}{n} = \boxed{\frac{e}{n} = \overline{HH_0}}$

Ex2 Circuit R-L-C.

1)



1] $t = 0^-$ $E(t) = 0$ $\Rightarrow$ $i(0^-) = 0$
C déchargé $\Rightarrow$ $u(0^-) = 0.$

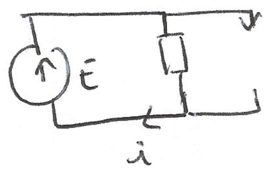
i traversant la bobine est continu $\Rightarrow$ $\boxed{i(0^+) = 0.}$
u aux bornes de C est continu $\Rightarrow$ $u(0^+) = 0.$

à $t = 0^+$ $E(t) = E$ et $E = u + u_L \rightarrow u_L(0^+) = E$

or $u_L(t) = L\frac{di}{dt}$ $\Rightarrow$ $\boxed{\frac{di}{dt}(0^+) = \frac{E}{L.}}$

2) En régime permanent: bobine $\Leftrightarrow$ fil ; condensateur $\Rightarrow$ interrupteur ouvert.

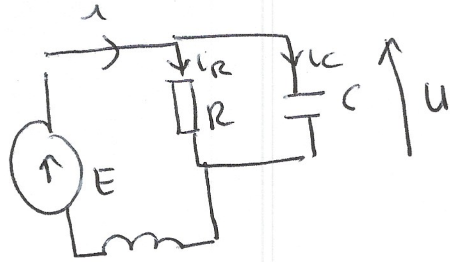
Le circuit devient



$i_C(\infty) = 0.$

$\boxed{i(\infty) = \frac{E}{R}}$

3°]



① $E = u + L\dfrac{di}{dt} \Rightarrow u = E - L\dfrac{di}{dt}$.

② $i = i_R + i_C = \dfrac{u}{R} + C\dfrac{du}{dt}$.

En remplaçant u ds l'éq. 2 $\Rightarrow i = \dfrac{1}{R}\left(E - L\dfrac{di}{dt}\right) + C\dfrac{d}{dt}\left[E - L\dfrac{di}{dt}\right]$

$$i = -\frac{L}{R}\frac{di}{dt} + \frac{E}{R} - LC\frac{d^2i}{dt^2} \Rightarrow LC\frac{d^2i}{dt^2} + \frac{L}{R}\frac{di}{dt} + i = \frac{E}{R}.$$

$$\boxed{\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{E/R}{LC}.}$$

$\boxed{S_p = \dfrac{E}{R} \text{ donc correspond bien à } i(\infty) \Rightarrow \text{cohérent}}$

$$\boxed{\frac{d^2i}{dt^2} + 2\lambda\omega_0\frac{di}{dt} + \omega_0^2 i = \frac{E/R}{LC}} \Rightarrow \boxed{\omega_0^2 = \frac{1}{LC}.}$$

$$2\lambda\omega_0 = \frac{1}{RC} \rightarrow \lambda = \frac{1}{2RC}\cdot\frac{\sqrt{LC}}{\sqrt{1}} \Rightarrow \boxed{\lambda = \frac{1}{2R}\sqrt{\frac{L}{C}}.}$$

$$\underbrace{\left[\frac{d^2i}{dt^2}\right]}_{IT^{-2}} = \underbrace{[2\lambda\omega_0]}_{T^{-1}}\underbrace{\left[\frac{di}{dt}\right]}_{IT^{-1}} = [\omega_0^2]\underbrace{[i]}_{I} \Rightarrow [\omega_0^2] = T^{-2} \Rightarrow \boxed{[\omega_0] = T^{-1}} \Rightarrow \boxed{[\lambda] = 1}$$

<u>Vérification</u> $\omega_0^2 = \dfrac{1}{LC}$ or $\left[\dfrac{L}{R}\right] = T$ et $[RC] = T \Rightarrow$

$[LC] = \left[\dfrac{L}{R}\right]\cdot[RC] = T^2$ donc $\boxed{[\omega_0] = T^{-1}}$

$\cancel{[2\lambda\omega_0] = T^{-1}}$ et $\left[\dfrac{L}{C}\right] = \dfrac{[R]\cdot T}{T\,[R^{-1}]} = [R]^2 \rightarrow \sqrt{\dfrac{L}{C}} = [R] \Rightarrow [\lambda] = 1$.

5°] $\lambda = 0{,}25$. $i(t) = S_{Go}(t) + \dfrac{E}{R}$ $S_{Go}(t)$ est solution

de $\dfrac{d^2i}{dt^2} + 2\lambda\omega_0\dfrac{di}{dt} + \omega_0^2 i(t) = 0$ e^{rt} solution $\Rightarrow r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$\Delta = 4\lambda^2\omega_0^2 - 4\omega_0^2 = 4\omega_0^2(\lambda^2 - 1) \Rightarrow r = -\lambda\omega_0 \pm i\omega_0\sqrt{1-\lambda^2}$

$$\Rightarrow \boxed{i(t) = Ae^{-\lambda\omega_0 t}\cos(\omega t + \varphi) + \frac{E}{R}.}$$

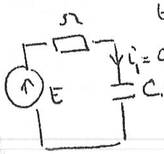
Champ magnétique intense.

$E = 30\,kV = 30.10^3\,V = 3.10^4\,V$

$C = 120\,\mu F = 120.10^{-6}\,F = 12.10^{-5}\,F$.

$L = 10\,nH = 10.10^{-9}\,H = 10^{-8}\,H$.

$r = 10\,m\Omega = 10.10^{-3}\,\Omega = 10^{-2}\,\Omega$.

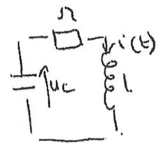
1] 

$t \to \infty$

$E = r i_1 + u_C$ avec $i(\infty) = 0$

$\Rightarrow \boxed{u_C(\infty) = E}$

2] $W_C = \frac{1}{2} C E^2$. AN $W_C = \frac{1}{2} . 12.10^{-5} . 9 . 10^8 = 54.10^3\,J \Rightarrow \boxed{W_C = 53\,kJ}$

3] 

$$\begin{cases} -u_C + r i + L \frac{di}{dt} = 0 \quad (1) \\ i = -C \frac{du_C}{dt} \quad (2) \end{cases} \Rightarrow \text{En dérivant (1).}$$

$$\frac{i}{C} + r \frac{di}{dt} + L \frac{d^2 i}{dt^2} = 0 \Rightarrow \boxed{\frac{d^2 i}{dt^2} + \frac{r}{L} \frac{di}{dt} + \frac{i}{LC} = 0}$$

A identifier à $\frac{d^2 i}{dt^2} + \frac{\omega_0}{Q} \frac{di}{dt} + \omega_0^2 i = 0 \rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$.

$\frac{\omega_0}{Q} = \frac{r}{L} \rightarrow Q = \frac{L \cdot \omega_0}{r} = \boxed{\frac{1}{r}\sqrt{\frac{L}{C}} = Q}$

AN $\omega_0 = \frac{1}{\sqrt{10^{-8} . 12.10^{-5}}} = \frac{1}{\sqrt{1,2 . 10^{-12}}} = \frac{10^6}{\sqrt{1,2}} = \boxed{9,1.10^5\,rad\,s^{-1}}$

$Q = \frac{1}{10^{-2}} \sqrt{\frac{10^{-8}}{12.10^{-5}}} = 10^2 . \frac{10^{-2}}{\sqrt{1,2}} = 0,91$

La solution de l'équation est $a\,e^{\alpha t} \Rightarrow \alpha^2 + \frac{\omega_0}{Q} \alpha + \omega_0^2 = 0$

$\Delta = \frac{\omega_0^2}{Q^2} - 4\omega_0^2 = \left(\frac{1}{Q^2} - 4\right)\omega_0^2 < 0 \Rightarrow$ régime pseudo périodique si $Q > \frac{1}{2}$.

or $Q = 0,91$. On est bien en régime pseudo périodique

$\boxed{i(0^+) = i(0^-) = 0.}$

La loi des mailles à $t \geq 0^+$ s'écrit $u_C = r i + L \frac{di}{dt}$.

or $u_C(0^-) = E$ et u_C est continue donc

à $t = 0^+$, On obtient. $E = 0 + L \frac{di}{dt}(0^+) \Rightarrow \boxed{\frac{di}{dt}(0^+) = \frac{E}{L}.}$

$i(t) = A e^{-\frac{\omega_0}{2Q} t} \cos(\omega t + \varphi)$ avec $\omega = \omega_0 \sqrt{1 - \frac{1}{4Q^2}}$.

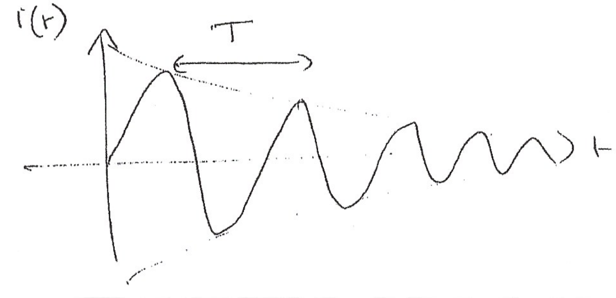
$\boxed{i(0^+) = 0 = A \cos \varphi} \Rightarrow \varphi = \frac{\pi}{2}$

$$\frac{di}{dt} = A e^{-\frac{\omega_0}{2Q} t} \left[-\frac{\omega_0}{2Q} \cos(\omega t + \varphi) - \omega \sin(\omega t + \varphi)\right]$$

$$\frac{di}{dt}(0^+) = A \left[-\frac{\omega_0}{2Q} \underbrace{\cos \varphi}_{= 0} - \omega \sin \varphi\right] = \frac{E}{L} \rightarrow A = -\frac{E}{L\omega} = \frac{-E}{L\omega_0 \sqrt{1 - \frac{1}{4Q^2}}}$$

$$i(t) = \frac{-E}{L\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} \cos\left(\omega t + \frac{\pi}{2}\right) \rightarrow \boxed{i(t) = \frac{E}{L\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} e^{-\frac{\omega_0 t}{2Q}} \sin \omega t}$$

Courbes enveloppes $\pm \frac{E}{L\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} e^{-\frac{\omega_0}{2Q} t}$



Durée de vie $\simeq 3\tau$

$e^{-\frac{\omega_0}{2Q} t} = e^{-\frac{t}{\tau}}$.

$\tau = \frac{2Q}{\omega_0} = \frac{2 . 0,91}{91.10^5} = 2.10^{-6}\,s$

$3\tau = \frac{6 . Q}{\omega_0} = 6.10^{-6}\,s$.

$\frac{1}{2} L i_{max}^2 = W_C \Rightarrow i_{max} = \sqrt{\frac{2 W_C}{L}} = \sqrt{\frac{2 \cdot 53 \cdot 10^{-3}}{10^{-8}}} = \sqrt{2 \cdot 5{,}3} \cdot 10^{12} = 33 \cdot 10^{6}\,A$ $= i_{max}$

$B = \frac{\mu_0 i}{D} = \frac{4\pi \cdot 10^{-7} \cdot 33 \cdot 10^{6}}{20 \cdot 10^{-3}} = \frac{4\pi \cdot 33}{2} \cdot 10^{2} = \boxed{207\,T = B}$

b] Dissipation d'énergie ds la résistance.

2] $U_C(t = 0^+) = E > 0$

$U_C(t) = A_1 e^{-\frac{\omega_0}{2Q} t} \cos(\omega t + \varphi_1)$

avec $\left.\begin{array}{l} U_C(0^+) = E \\ \text{et } \frac{dU_C}{dt}(0^+) = \frac{i(0^+)}{C} = 0 \end{array}\right\} \Rightarrow \begin{array}{l} A_1 \cos\varphi_1 = E \\ A_1 e^{-\frac{\omega_0}{2Q} t} \left[-\frac{\omega_0}{2Q} \cos(\omega t + \varphi_1) - \omega \sin(\omega t + \varphi_1) \right] = \frac{dU_C}{dt} \end{array}$

d'où $-\frac{\omega_0}{2Q} \cos\varphi_1 - \omega \sin\varphi_1 = 0 \rightarrow \tan\varphi_1 = \frac{\sin\varphi_1}{\cos\varphi_1} = \frac{-\omega_0}{2Q\,\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} = -\frac{1}{\sqrt{4Q^2 - 1}}$.

et $\cos^2\varphi_1 = \frac{1}{1 + \tan^2\varphi_1} = \frac{1}{1 + \frac{1}{4Q^2 - 1}} = \frac{4Q^2 - 1}{4Q^2} = 1 - \frac{1}{4Q^2}$.

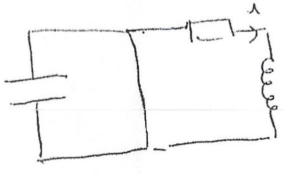
$\Rightarrow \cos\varphi_1 = \sqrt{1 - \frac{1}{4Q^2}}$

d'où $U_C(t) = \frac{E}{\sqrt{1 - \frac{1}{4Q^2}}} \cdot e^{-\frac{\omega_0}{2Q} t} \cos(\omega t + \varphi_1)$.

$U_C(t) = 0$ pour $t = t_0$ vérifiant $\omega t_0 + \varphi_1 = \frac{\pi}{2} \rightarrow t_0 = \frac{\frac{\pi}{2} - \varphi_1}{\omega_0 \sqrt{1 - \frac{1}{4Q^2}}} = 3 \cdot 10^{-6}\,s$.

avec $\varphi_1 = -46° = -\left(\frac{46 \cdot \pi}{180}\right)$ rad.

3] $t > t_0$ la diode est débloquée



$ri + L\frac{di}{dt} = 0 \rightarrow i(t) = A e^{-\frac{t}{\tau}}$ avec $\tau = \frac{L}{r}$

$\frac{di}{dt} + \frac{r}{L} i = 0$

$i(t) = A e^{-\frac{t}{\tau}}$

$i(t_0) = A e^{-\frac{t_0}{\tau}} \Rightarrow A = \frac{i(t_0)}{e^{-\frac{t_0}{\tau}}}$.

d'où $\boxed{i(t) = i(t_0)\, e^{-\frac{(t - t_0)}{\tau}}}$.

$i(t) = i(t_0) \cdot \frac{10}{100} = \frac{i(t_0)}{10} = i(t_0)\, e^{-\frac{(t - t_0)}{\tau}}$ avec $t - t_0 = \tau'$

$\frac{1}{10} = e^{-\frac{\tau'}{\tau}} \Rightarrow -\ln 10 = -\frac{\tau'}{\tau} \rightarrow \tau' = \tau \ln 10 = \frac{10^{-8}}{10^{-2}} \ln 10$ | 2,31