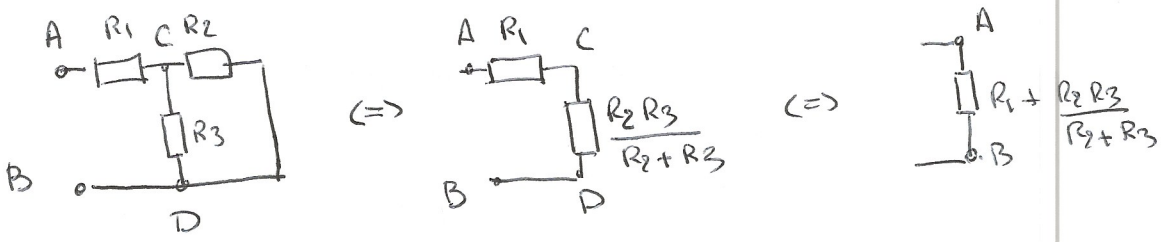
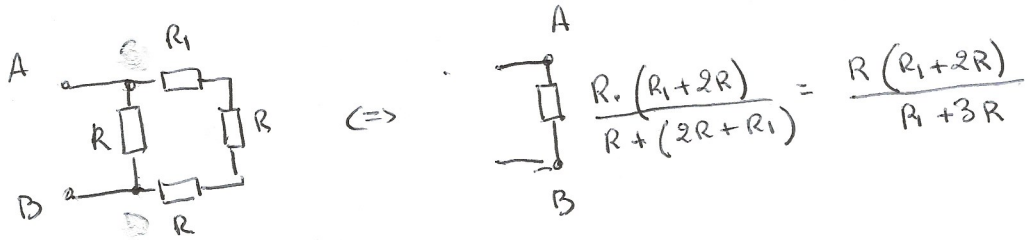
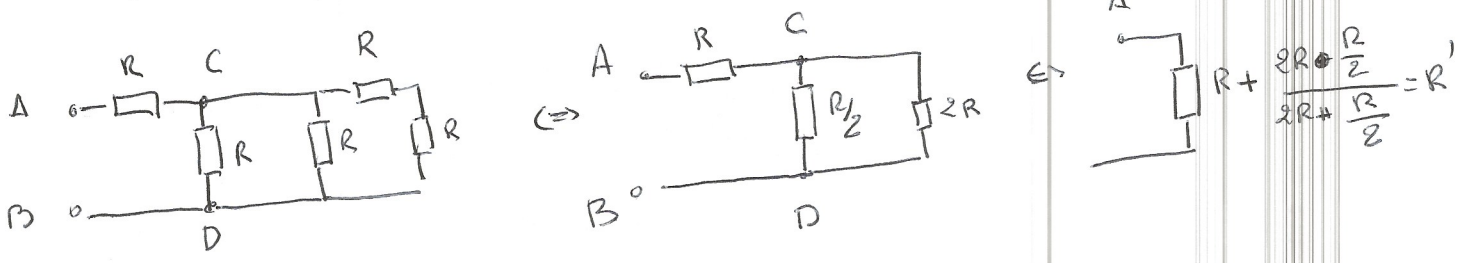
Correction TD Electrocinétique - 1

Ex1 Association de résistances.

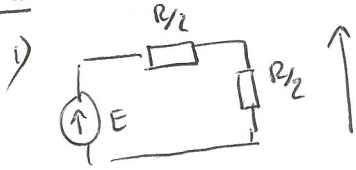




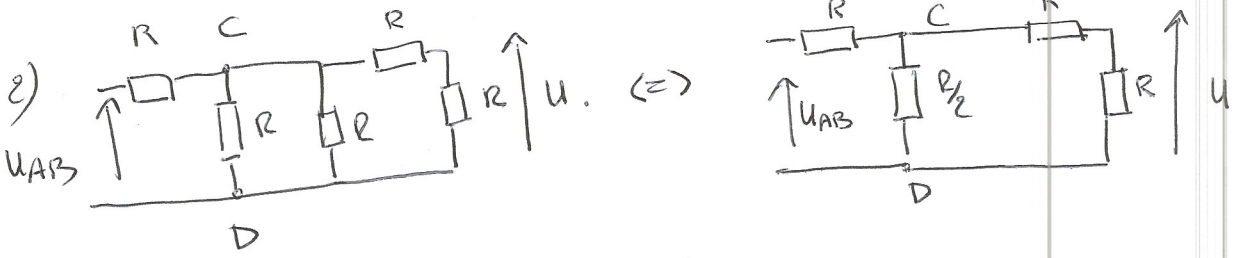


d'où $R' = R + \frac{2R^2}{5R} = R + \frac{2}{5}R = \frac{7}{5}R$

Ex2 Diviseur de tension.

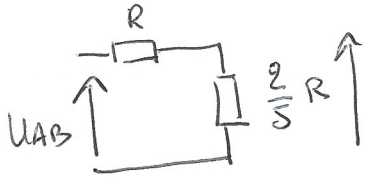
1) 

$U = \frac{\frac{R}{2}}{\frac{R}{2} + \frac{R}{2}} \cdot E \Rightarrow \boxed{U = \frac{E}{2}}$

2) 

$U = \frac{R}{2R} U_{CD} \Rightarrow U = \frac{U_{CD}}{2}$.

On applique le diviseur de tension une 2ème fois :

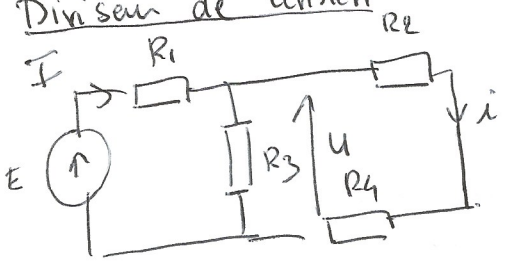


$U_{CD} = \frac{\frac{2}{5}R}{R + \frac{2}{5}R} \cdot U_{AB} = \frac{2}{7}U_{AB}$.

d'où $U = \frac{U_{CD}}{2} = \frac{2}{7} \cdot U_{AB} \cdot \frac{1}{2} \Rightarrow \boxed{U = \frac{U_{AB}}{7}}$.

Ex3 Comparaison diviseur de tension - diviseur de courant.

Diviseur de tension :



$$U = \frac{\frac{R_3(R_2+R_4)}{R_3+R_2+R_4}}{R_1 + \frac{R_3(R_2+R_4)}{R_2+R_3+R_4}} \cdot E = \frac{R_3(R_2+R_4)}{R_1(R_2+R_3+R_4)+R_3(R_2+R_4)}$$

$$i = \frac{U}{R_2+R_4} = \boxed{\frac{R_3}{R_1(R_2+R_3+R_4)+R_3(R_2+R_4)} \cdot E = i}$$

Diviseur de courant :

$$I = \frac{E}{R_1 + \frac{R_3(R_2+R_4)}{R_2+R_3+R_4}} = \frac{E(R_2+R_3+R_4)}{R_1(R_2+R_3+R_4)+R_3(R_2+R_4)}$$

$$\frac{i}{I} = \frac{R_3(R_2+R_4)/R_2+R_3+R_4}{R_2+R_4} = \frac{R_3}{R_2+R_3+R_4}$$

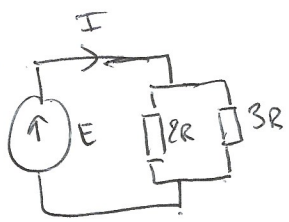
d'où $\boxed{i = \frac{R_3 \cdot E}{R_1(R_2+R_3+R_4)+R_3(R_2+R_4)}}$

On retrouve bien la même chose.

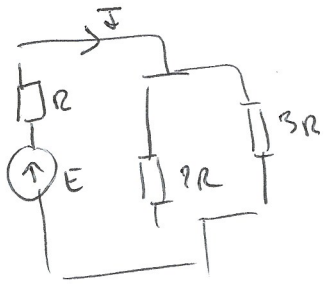
AN $U = \frac{50 \cdot 100 \cdot 6}{100 \cdot 150 + 50 \cdot 100} = \frac{50}{200} \cdot 6 = \frac{6}{4} = \frac{3}{2} = \boxed{1{,}5\,V = U}$

$\boxed{i = \frac{1{,}5}{100} = 15\,mA}$

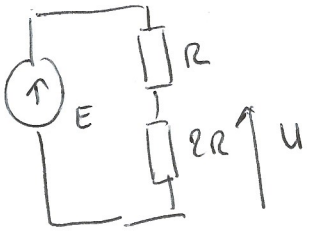
Ex4



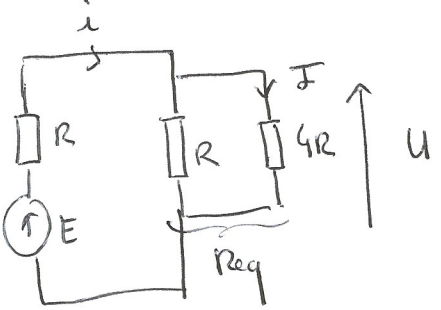
$$R_{eq} = \frac{2R \cdot 3R}{5R} = \frac{6}{5}R \qquad I = \frac{E}{\frac{6}{5}R} = \frac{5}{6} \cdot \frac{E}{R}$$



$$I = \frac{E}{R + \frac{6}{5}R} = \frac{5}{11} \cdot \frac{E}{R}$$



$$\frac{U}{E} = \frac{2R}{3R} \Rightarrow U = \frac{2}{3}E$$



$J = \frac{U}{4R}$ et $\frac{U}{E} = \frac{Req}{R+Req}$ avec $Req = \frac{4R^2}{5R} = \frac{4}{5}R$.

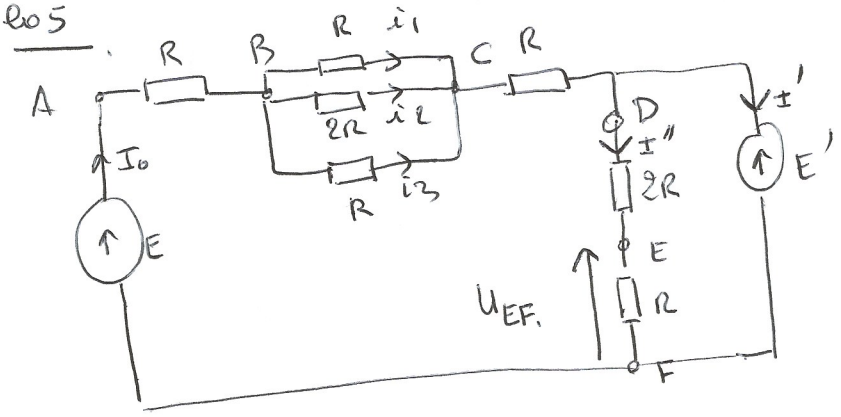
d'où $U = \frac{\frac{4}{5}R}{R+\frac{4}{5}R} \cdot E = \frac{4}{9}E$.

et $J = \frac{E}{9R}$.

ou $i = \frac{E}{R+Req} = \frac{E}{R+\frac{4}{5}R} = \frac{5}{9}\cdot\frac{E}{R}$ et $\frac{J}{i} = \frac{Req}{4R} = \frac{1}{5}$

d'où $\boxed{I = \frac{E}{9R}}$

Pb 5



1] $\frac{U_{EF}}{E'} = \frac{R}{3R} \rightarrow \boxed{U_{EF} = \frac{E'}{3}}$.

2] $\frac{1}{R_{BC}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{2R} = \frac{2}{R} + \frac{1}{2R}$.

$= \frac{4+1}{2R} = \frac{5}{2R} \rightarrow \boxed{R_{BC} = \frac{2R}{5}}$

Loi des mailles $-E + (R + R_{BC} + R)I_0 + E' = 0$.

$I_0 = \frac{E-E'}{2R+\frac{2R}{5}} = \boxed{\frac{5(E-E')}{12R} = I_0}$.

3] $I' = I_0 - I''$ et $I'' = \frac{E'}{3R}$. $\rightarrow I' = \frac{5E-5E'}{12R} - \frac{E'}{3R} = \frac{5E-5E'-4E'}{12R}$.

$\boxed{I' = \frac{5E-9E'}{12R}}$

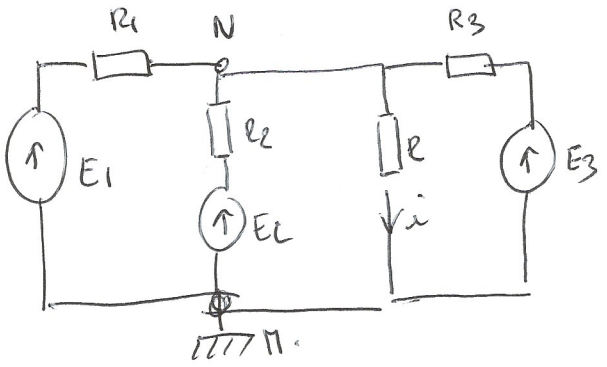
4°] Diviseur de courant. $\frac{i_1}{I_0} = \frac{R_{BC}}{R} \rightarrow \boxed{i_1 = \frac{2}{5}I_0}$ $\boxed{i_3 = i_1 = \frac{2}{5}I_0}$.

$\frac{i_2}{I_0} = \frac{R_{BC}}{2R} \rightarrow \boxed{i_2 = \frac{1}{5}I_0}$

Rq La loi des nœuds est bien vérifiée

Ex6 : loi des noeuds en termes de potentiels.

a)



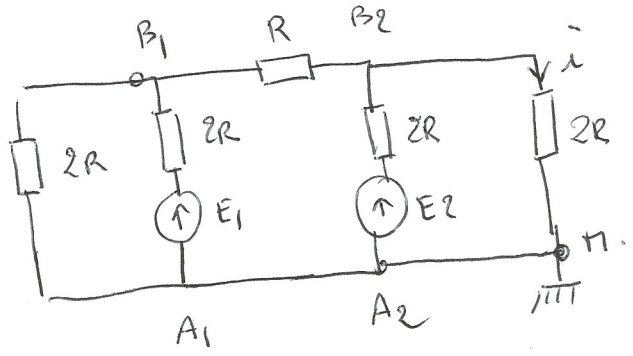
$i = \dfrac{V_N - V_M}{R}$ $\quad V_N$: potentiel en N, V_M " en M.

On fixe $V_M = 0$.

$$V_N = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + 0 + \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R} + \frac{1}{R_3}} = \frac{\frac{R_2R_3E_1 + R_1R_3E_2 + R_1R_2E_3}{R_1R_2R_3}}{\frac{R_2RR_3 + R_1RR_3 + R_1R_2R_3 + R_1R_2R}{RR_1R_2R_3}}$$

$$\boxed{V_N = \frac{R\left[R_2R_3E_1 + R_1R_3E_2 + R_1R_2E_3\right]}{R(R_2R_3 + R_1R_3 + R_1R_2) + R_1R_2R_3}}$$

b)



$V_{A_1} = V_{A_2} = V_M = 0$.

$i = \dfrac{V_{B_2} - V_M}{2R} = \dfrac{V_{B_2}}{2R}$.

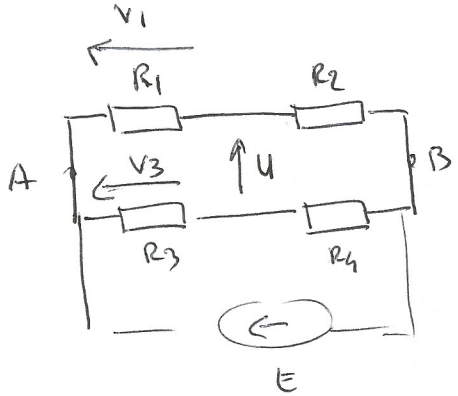
$$V_{B_2} = \frac{\frac{V_{B_1}}{R} + \frac{E_2}{2R} + \frac{V_M}{2R}}{\frac{1}{R} + \frac{1}{2R} + \frac{1}{2R}} = \frac{\left(2V_{B_1} + E_2\right)/2R}{4/2R} \Rightarrow \boxed{4V_{B_2} = 2V_{B_1} + E_2} \quad (1)$$

$$V_{B_1} = \frac{\frac{0}{2R} + \frac{V_{B_2}}{R} + \frac{E_1}{2R}}{\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}} = \frac{\left(2V_{B_2} + E_1\right)/2R}{4/2R} \rightarrow \boxed{4V_{B_1} = 2V_{B_2} + E_1} \quad (2)$$

On a alors $2\cdot(1) \Rightarrow 8V_{B_2} = 4V_{B_1} + 2E_2 = 2V_{B_2} + E_1 + 2E_2$

d'où $\boxed{V_{B_2} = \dfrac{E_1 + 2E_2}{6}}$ et $\boxed{i = \dfrac{E_1 + 2E_2}{12R}}$

Ex 7 Pont de Wheatstone



$$\frac{V_1}{E} = \frac{R_1}{R_1+R_2} \Rightarrow \begin{cases} \text{loi des mailles} \\ V_1 + U - V_3 = 0 \\ U = V_3 - V_1 \end{cases}$$

$$\frac{V_3}{E} = \frac{R_3}{R_3+R_4}.$$

d'où $U = \left(\frac{R_3}{R_3+R_4} - \frac{R_1}{R_1+R_2} \right) E = \frac{(R_1R_3 + R_2R_3 - R_1R_3 - R_4R_1) \cdot E}{(R_3+R_4)(R_1+R_2)}$

$$\boxed{U = \frac{R_2R_3 - R_4R_1}{(R_3+R_4)(R_1+R_2)} \cdot E}$$

$U = 0 \Rightarrow R_2R_3 = R_4R_1$

$$\rightarrow \boxed{R_1 = \frac{R_2R_3}{R_4}}$$