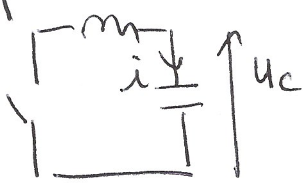
Correction TD circuits 2nd ordre

en régime transitoire

Ex1



1) à $t=0^-$ $i=0$ (K ouvert) $\Rightarrow$ $i(0^+)=0$

$u=u_0$ (C chargé) $\Rightarrow$ $u(0^+)=u_0$.

car i traversant la bobine et u tension aux bornes de C sont continus.

2) $\begin{cases} L\dfrac{di}{dt}+u_C=0 \\ i=C\dfrac{du_C}{dt} \end{cases} \rightarrow LC\dfrac{d^2u_C}{dt^2}+u_C=0 \qquad \dfrac{d^2u_C}{dt^2}+\dfrac{1}{LC}u_C=0$

à identifier à $\dfrac{d^2u_C}{dt^2}+\omega_0^2u_C=0$ $\Rightarrow \omega_0^2=\dfrac{1}{LC}$

$u_C(t) = A\cos(\omega_0 t+\varphi)$ et $u_C(0^+)=u_0=A\cos\varphi$

$i(0^+)=C\dfrac{du_C}{dt}(0^+)=0=-A\omega_0\sin\varphi \Rightarrow \varphi=0.$

d'où $\boxed{u_C(t)=u_0\cos\omega_0 t}$

3°) $i=C\dfrac{du_C}{dt}=-C\omega_0u_0\sin\omega_0t=-C\cdot\dfrac{u_0}{\sqrt{LC}}\sin\omega_0t=-\sqrt{\dfrac{C}{L}}\cdot u_0\sin\omega_0 t$

$\boxed{i(t)=\sqrt{\dfrac{C}{L}}\,u_0\cos\left(\omega_0t+\dfrac{\pi}{2}\right)}$ Rq $\left[\sqrt{\dfrac{C}{L}}\right]=\sqrt{\dfrac{[C]}{[L]}}=\sqrt{\dfrac{T/[R]}{T\cdot[R]}}=\dfrac{1}{[R]}$

donc $\left[\sqrt{\dfrac{C}{L}}\cdot u_0\right]=\dfrac{[\text{Tension}]}{[\text{Resistance}]}$

$u_L(t)=L\dfrac{di}{dt}=-L\sqrt{\dfrac{C}{L}}\cdot u_0\cdot\dfrac{1}{\sqrt{LC}}\cos\omega_0t=\boxed{-u_0\cos\omega_0t=-u_C(t)}$ (prévisible car $u_C+u_L=0$)

4°) $(u_L+u_C)\cdot i=0$

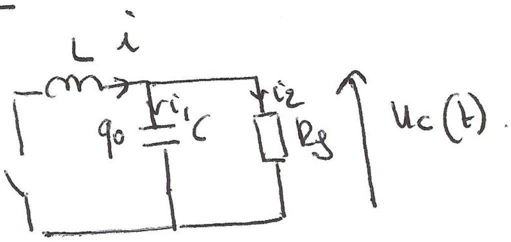
$P_C+P_L=0$

$\dfrac{d}{dt}(\mathcal{E}_C+\mathcal{E}_L)=0 \Rightarrow \mathcal{E}_C+\mathcal{E}_L=\dfrac{1}{2}Cu_C^2(t)+\dfrac{1}{2}Li^2(t)=\text{cste}=\mathcal{E}_C(t=0)+\mathcal{E}_L(t=0)$

$=\dfrac{1}{2}Cu_0^2$

$$\boxed{\frac{1}{2}Cu_C^2(t)+\frac{1}{2}L\,i^2(t)=\frac{1}{2}Cu_0^2}$$

Ex3



$$\begin{cases} L\dfrac{di}{dt} + u_c = 0 \quad (1) \\ i_1 = C\dfrac{du_c}{dt} \quad (2) \quad \text{et} \quad i = i_1 + i_2 \quad (4) \\ i_2 = \dfrac{u_c}{R}. \quad (3) \end{cases}$$

(4) ds (1) $\Rightarrow$ $L\dfrac{di_1}{dt} + L\dfrac{di_2}{dt} + u_c = 0$ A l'aide de 2 et 3

$LC\dfrac{d^2u_c}{dt^2} + \dfrac{L}{R}\dfrac{du_c}{dt} + u_c = 0$ $\Rightarrow$ $\boxed{\dfrac{d^2u_c}{dt^2} + \dfrac{1}{RC}\dfrac{du_c}{dt} + \dfrac{u_c}{LC} = 0.}$

A comparer à la forme canonique $\dfrac{d^2u_c}{dt^2} + 2\lambda\omega_0\dfrac{du_c}{dt} + \omega_0^2 u_c = 0$

ou $\dfrac{d^2u_c}{dt^2} + \dfrac{\omega_0}{Q}\dfrac{du_c}{dt} + \omega_0^2 u_c = 0.$

donc $\boxed{\omega_0^2 = \dfrac{1}{LC}}$ et $\dfrac{\omega_0}{Q} = \dfrac{1}{RC}$ $\to$ $Q = \omega_0 \cdot RC = \dfrac{1}{\sqrt{LC}} \cdot RC$

$\boxed{\omega_0 = \dfrac{1}{\sqrt{LC}}}$ et $\boxed{Q = R \cdot \sqrt{\dfrac{C}{L}}}$

2°] à $t = 0^-$ $q(0^-) = q_0 = q(0^+)$ et $u(0^+) = \dfrac{q(0^+)}{C} = \dfrac{q_0}{C}.$ (Continuité de u donc de q aux bornes de C)

$i(0^-) = i(0^+)$ et $i(0^-) = 0$ (K ouvert)

continuité de i traversant la bobine. $\Rightarrow$ $i(0^+) = 0.$

$i_2(0^+) = \dfrac{u_c(0^+)}{Rg} = \dfrac{q_0}{Rg \cdot C}.$

d'où $i_1(0^+) = i(0^+) - i_2(0^+) = -\dfrac{q_0}{Rg \cdot C} = C\dfrac{du_c(0^+)}{dt}$

$\Rightarrow$ $\dfrac{du_c(0^+)}{dt} = -\dfrac{q_0}{C} \cdot \dfrac{1}{Rg \cdot C} = \boxed{-\dfrac{q_0}{Rg\, C^2} = \dfrac{du_c}{dt}(0^+)}$

3°] Régime pseudo périodique $u_c(t) = Ae^{-\frac{\omega_0}{2Q}t}\cos(\omega t + \varphi)$

avec $\omega = \omega_0\sqrt{1 - \dfrac{1}{4Q^2}}$

$$u_C(0^+) = \frac{q_0}{C} = A\cos\varphi.$$

$$\frac{du_C}{dt} = A\left[-\frac{\omega_0}{2Q} e^{-\frac{\omega_0}{2Q}t}\cos(\omega t+\varphi) - \omega e^{-\frac{\omega_0}{2Q}t}\sin(\omega t+\varphi)\right]$$

$$\frac{du_C}{dt}(0^+) = A\left[-\frac{\omega_0}{2Q}\cos\varphi - \omega\sin\varphi\right] = -\frac{q_0}{R_g\cdot C^2}$$

$$\frac{\cancel{q_0}}{C}\left[-\frac{\omega_0}{2Q} - \omega\,\mathrm{tg}\,\varphi\right] = -\frac{\cancel{q_0}}{R_g\cdot C^2}$$

$$\frac{\omega_0}{2Q} + \omega\,\mathrm{tg}\,\varphi = \frac{1}{R_g\cdot C} \rightarrow \boxed{\mathrm{tg}\,\varphi = \frac{\frac{\omega_0}{2Q} - \frac{1}{R_g\cdot C}}{\omega}}$$

et $\frac{1}{\cos^2\varphi} = 1+\mathrm{tg}^2\varphi \Rightarrow \frac{1}{\cos\varphi} = \sqrt{1+\left(\frac{\omega_0}{2Q\omega} - \frac{1}{R_g\cdot C\cdot\omega}\right)^2}$ $\left(\begin{array}{l}\varphi = \mathrm{Arctg}(\ \ )\\ \cos\varphi > 0\end{array}\right)$

$$\text{d'où } A = \frac{q_0}{C}\cdot\frac{1}{\cos\varphi}.$$

$$u_C(t) = \frac{q_0}{C}\cdot\sqrt{1+\left(\frac{\omega_0}{2Q\omega} - \frac{1}{R_g\cdot C\omega}\right)^2}\cos(\omega t+\varphi).$$

$u_C(t) = 0{,}26\,V$ (1er sommet)

$u_C(t+3T) = 0{,}06\,V$

4°] $\delta = \frac{1}{n}\ln\frac{u_C(t)}{u_C(t+nT)}$

$$\Rightarrow \delta = \frac{1}{3}\ln\frac{u_C(t)}{u_C(t+3T)} = \boxed{0{,}49 = \delta}$$

or $\delta = \frac{1}{n}\ln\left(\frac{A e^{-\frac{\omega_0}{2Q}t}\cos(\omega t+\varphi)}{A e^{-\frac{\omega_0}{2Q}(t+nT)}\underbrace{\cos(\omega(t+nT)+\varphi)}_{\parallel\ \cos\omega t+\varphi}}\right) = \frac{1}{n}\ln\exp\left(\frac{\omega_0}{2Q}\cdot nT\right)$

$$= \frac{\cancel{\omega_0}}{2Q}\cdot\frac{2\pi}{\cancel{\omega_0}\sqrt{1-\frac{1}{4Q^2}}} = \boxed{\frac{2\pi}{\sqrt{4Q^2-1}} = \delta} \Rightarrow \frac{4\pi^2}{4Q^2-1} = \delta^2$$

$$4Q^2 = \frac{4\pi^2}{\delta^2} + 1$$

$$\boxed{Q^2 = \frac{\pi^2}{\delta^2} + \frac{1}{4}} \Rightarrow \boxed{Q = 6,4}$$

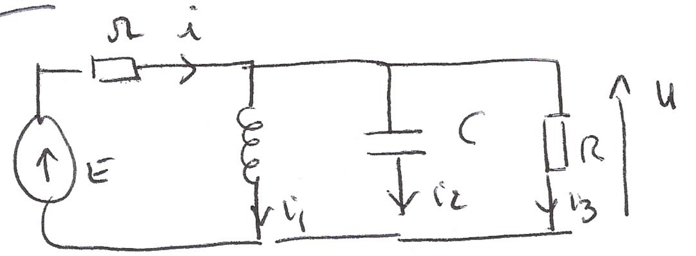
$$T = \frac{2\pi}{\omega_0\sqrt{1 - \frac{1}{4Q^2}}} = 4 \times 0,2 \cdot 10^{-5}\,s = 0,8 \cdot 10^{-5}\,s \Rightarrow \omega_0 = 7,87 \cdot 10^5\,rad\,s^{-1}$$

or $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 \cdot C} = \frac{1}{(7,87)^2 \cdot 10^{10} \cdot 5 \cdot 10^{-9}} = 3,2 \cdot 10^{-4}\,H$

$$\boxed{L = 32\,mH}$$

Enfin $Q = \frac{R}{S}\sqrt{\frac{L}{C}} \Rightarrow \boxed{R_g = 1,6 \cdot 10^3\,\Omega}$

Ex4



$$\begin{cases} E = ri + u. & (1) \\ u = L\dfrac{di_1}{dt} = Ri_3 & (2) \\ i_2 = C\dfrac{du}{dt} & (3) \\ i = i_1 + i_2 + i_3 & (4) \end{cases}$$

1°/ à $t = 0^-$ $\quad i_1(0^-) = 0 \rightarrow i_1(0^+) = 0$

$u(0^-) = 0 \rightarrow u(0^+) = 0.$

(1) $\quad i(0^+) = \dfrac{E}{r}.$

(2) $\quad i_3(0^+) = 0$

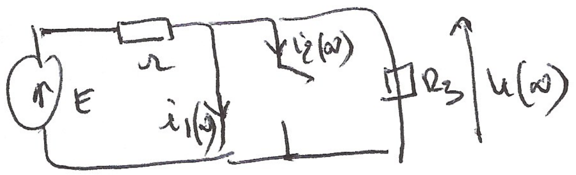
(4) $\Rightarrow i_2(0^+) = i(0^+) = \dfrac{E}{r}$

Rq $\quad \dfrac{di_3}{dt}(0^+) = \dfrac{1}{R}\dfrac{du}{dt}(0^+)$

$= \dfrac{1}{RC}\, i_2(0^+) = \dfrac{E}{r.R}$

2°/ Regime permanent continu $\quad$ -m- ⟺ — $\quad$ -||- ⟺ -/-

Circuit équivalent.



$i_2(\infty) = 0$ (C = interrupteur ouvert)

$u(\infty) = 0$ (tension aux bornes d'un fil = 0)

$i_3(\infty) = 0$

$i_1(\infty) = i(\infty) = \dfrac{E}{r}$

3°] $u = Ri_3$.

or $E = r(i_1 + i_2 + i_3) + u.$

$E = r\left(i_1 + C\dfrac{du}{dt} + i_3\right) + u. \rightarrow$ On derive et remplace u par Ri_3

$0 = r\left[\dfrac{Ri_3}{L} + RC\dfrac{d^2i_3}{dt} + \dfrac{di_3}{dt}\right] + R\dfrac{di_3}{dt}$

$RC\dfrac{d^2i_3}{dt^2} + \left(1 + \dfrac{R}{r}\right)\dfrac{di_3}{dt} + \dfrac{R}{L}i_3 = 0 \quad \Rightarrow \dfrac{d^2i_3}{dt^2} + \dfrac{\left(1+\frac{R}{r}\right)}{RC}\dfrac{di_3}{dt} + \dfrac{i_3}{LC} =$

$$\boxed{\omega_0^2 = \frac{1}{LC}} \qquad 2\lambda\omega_0 = \frac{\left(1+\frac{R}{r}\right)}{R\cdot C} \Rightarrow \lambda = \frac{1+\frac{R}{r}}{RC} \cdot \frac{\sqrt{LC}}{2}.$$

$$\boxed{\lambda = \frac{1+\frac{R}{r}}{2R} \cdot \sqrt{\frac{L}{C}}.}$$

$$\frac{d^2 i_3}{dt^2} + 2\lambda\omega_0 \frac{di_3}{dt} + \omega_0^2 i_3 = 0$$

e^{rt} solution $\rightarrow$ $r^2 + 2\lambda\omega_0 r + \omega_0^2 = 0$

$\Delta = (4\lambda^2 - 4)\omega_0^2 < 0$ régime pseudo-périodique

$\Rightarrow \lambda < 1.$

$$\boxed{\frac{1+\frac{R}{r}}{2R}\sqrt{\frac{L}{C}} < 1}$$

5] $T = \dfrac{2\pi}{\omega_0\sqrt{1-\lambda^2}} = \boxed{\dfrac{2\pi \cdot \sqrt{LC}}{\sqrt{1 - \frac{(1+R/r)^2}{4R^2}\cdot\frac{L}{C}}} = T.}$

$$i_3(t) = Ae^{-\lambda\omega_0 t}\left(\cos(\omega t + \varphi)\right).$$

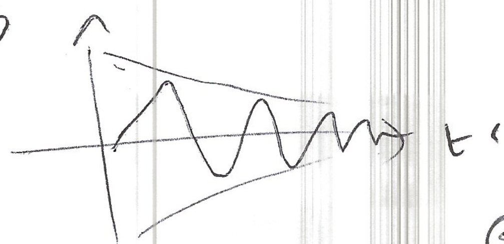
$i_3(0^+) = 0 \Rightarrow 0 = A\cos\varphi \Rightarrow \varphi = \frac{\pi}{2}.$

$$\frac{di_3}{dt} = A\left[\cancel{\omega}\ \lambda\omega_0 e^{-\lambda\omega_0 t}\cos(\quad) - \omega e^{-\lambda\omega_0 t}\sin(\quad)\right]$$

$$\frac{di_3}{dt}(0^+) = A\left[-\lambda\omega_0 \underbrace{\cos\varphi}_{0} - \omega \underbrace{\sin\varphi}_{1}\right] = \frac{E}{R\cdot rC}.$$

$$\Rightarrow A = -\frac{E}{R\cdot r\cdot C\omega}$$

6)



$$\Rightarrow \boxed{i_3(t) = \frac{E}{R\cdot rC\omega}e^{-\lambda\omega_0 t}\ \sin\omega t.}$$