

Calcul de $F = -\rho \int_0^R \int_0^{\pi/2} r^2 \cos\theta \sin\theta r dr d\theta$

$$F = -\rho \frac{R^4}{4} \int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta = -\rho \frac{R^4}{4} \left[-\frac{\cos 2\theta}{4} \right]_0^{\pi/2}$$

$$F = -\rho \frac{R^4}{8} \quad \text{or} \quad \rho = \frac{m}{\frac{\pi R^2}{4}} \Rightarrow \boxed{F = -\frac{m R^2}{2\pi}}$$

on retrouve bien

$$I_0(s) = \frac{m R^2}{2} \begin{pmatrix} 1/2 & -1/4 & 0 \\ \triangle & 1/2 & 0 \\ & & 1 \end{pmatrix}$$