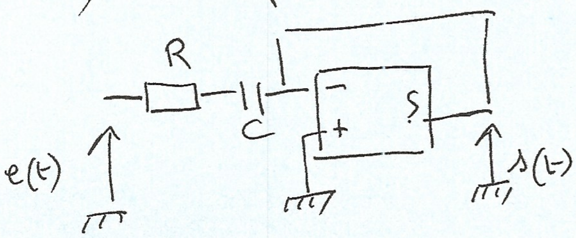
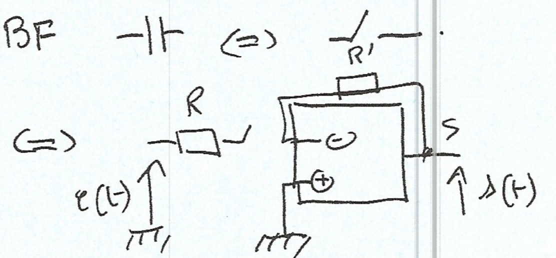
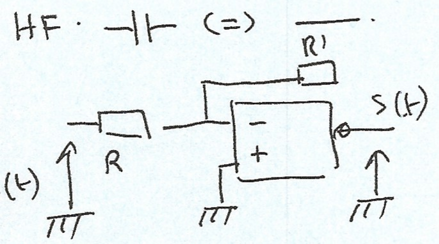
Correction TD ALi.

Ex1 1) Filtre passe haut du 1er ordre.





$V^+ = 0 = V^-$ or $V^- = s \Rightarrow s = 0$
car $i = 0$



$$V^{\ominus} = \frac{\frac{e}{R} + \frac{s}{R'}}{\frac{1}{R} + \frac{1}{R'}} = V^+ = 0 \rightarrow \boxed{s = -\frac{R'}{R} e}$$

2) Calcul de $\underline{H}$.

$$V^{\ominus} = \frac{\frac{e}{R + \frac{1}{jC\omega}} + \frac{s}{R'}}{\frac{1}{R + \frac{1}{jC\omega}} + \frac{1}{R'}} = \quad V^+ = 0 \quad \rightarrow \quad \frac{\underline{s}}{\underline{e}} = \frac{\ominus R'}{R + \frac{1}{jC\omega}} = \frac{-jR'C\omega}{1 + jRC\omega}.$$

$$\underline{H} = \frac{H_\infty \cdot j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}} \Rightarrow \begin{cases} \frac{1}{\omega_c} = RC \Rightarrow \omega_c = \frac{1}{RC}. \\ \frac{H_\infty}{\omega_c} = -R'C \Rightarrow \boxed{H_\infty = -\frac{R'}{R}.} \end{cases}$$

3°) AN $R = 1k\Omega$ et $\omega_c = 10^4 \text{ rad s}^{-1}$ $\rightarrow$ $C = \frac{1}{R\omega_c} = \frac{1}{10^3 \cdot 10^4} = \boxed{10^{-7} F = 0{,}1 \mu F = C}$

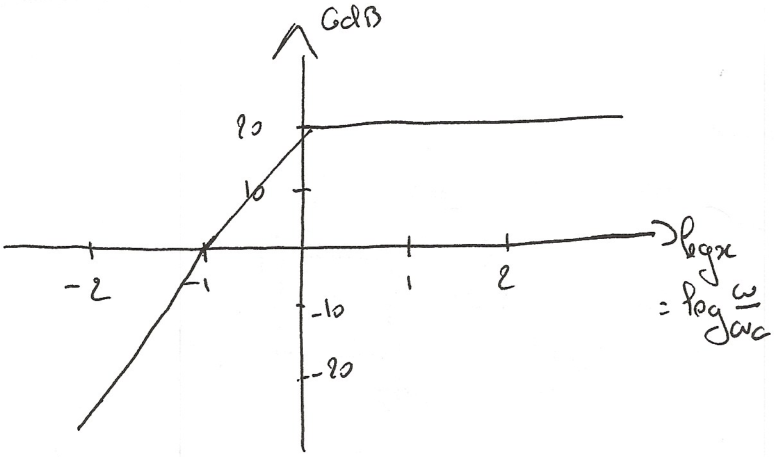
$G_{dB}(\infty) = 20 \log |H_\infty| = 20 \log \frac{R'}{R} = 20 \text{ dB.} \Rightarrow \frac{R'}{R} = 10 \rightarrow \boxed{R' = 10k\Omega}$

4°) $\underline{H} \xrightarrow{BF} H_\infty \cdot j\frac{\omega}{\omega_c} \rightarrow G_{dB} \Rightarrow 20 \log |H_\infty| + 20 \log x = Y_{BF}$ $x = \frac{\omega}{\omega_c}$

$\underline{H} \xrightarrow{HF} H_\infty \rightarrow G_{dB} \rightarrow 20 \log |H_\infty| = 20 \text{dB}$

φ varie de $-\frac{\pi}{2}$ à $-\pi$

$\varphi = -\frac{\pi}{2} - \arctan x.$



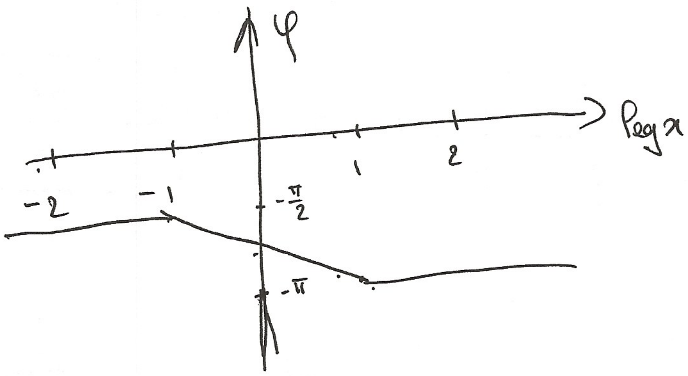
$\omega_c = 10^4 \text{ rad s}^{-1}$

$\omega_1 = 10^2 \text{ rad s}^{-1} \to x_1 = 10^{-2} \to G_1 = -20 \text{ dB} \quad \varphi_1 = -\frac{\pi}{2}$

$\omega_2 = 10^3 \text{ rad s}^{-1} \to x_2 = 10^{-1} \to G_2 = 0 \quad \varphi_2 = -\frac{\pi}{2}$

$\omega_3 = 10^5 \text{ rad s}^{-1} \to x_3 = 10 \to G_3 = 20 \text{dB} \quad \varphi_3 =$

$G_{dB} = 20 \log|H| \to |H| = 10^{\frac{G_{dB}}{20}}.$



$e_1(t) = 1 \cdot \cos \omega_1 t \to s_1(t) = 0{,}1 \cos(\omega_1 t - \frac{\pi}{2})$

$e_2(t) = 3 \cos \omega_2 t \to s_2(t) = 3 \cos(\omega_2 t - \frac{\pi}{2}$

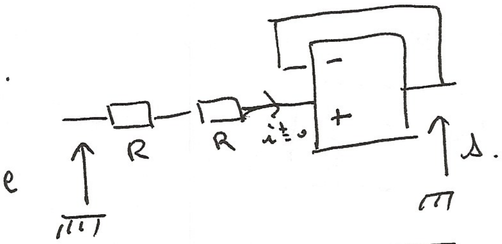
$e_3(t) = 1 \cos \omega_3 t \to s_3(t) = 10 \cos(\omega_3 t -$

<u>exo 2</u>

1] BF –||– ⇔ –/–

HF –||– ⇔ —.

BF.

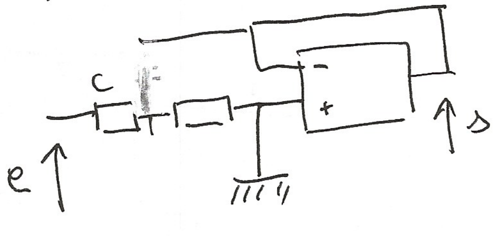


$e - v^+ = 2R \underset{0}{\underset{\|}{i^+}} = 0 \to v^+ = e$

$v^+ = v^- = s$

$\Big\} \; e = s.$

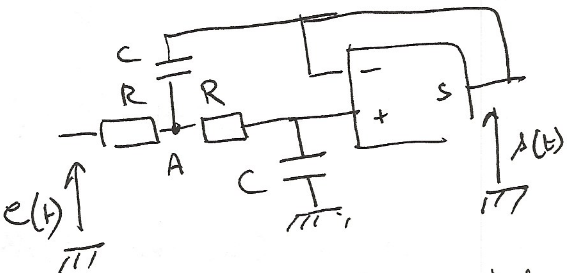
HF



$\left.\begin{array}{l} v^+ = 0 \\ v^- = s \end{array}\right\} \to s = 0.$

Filtre passe bas.

2°] Calcul de $\underline{H}$



$V^- = s$

$V^+ = \dfrac{\frac{V_A}{R}}{\frac{1}{R} + jC\omega} = \dfrac{V_A}{1 + jRC\omega} = s.$

$$V_A = \frac{\frac{e}{R} + jC\omega s + \frac{v^+}{R}}{\frac{1}{R} + jC\omega + \frac{1}{R}} = \frac{e + (1 + jRC\omega)s}{2 + jRC\omega}$$ puisque $v^+ = s$.

En égalant les 2 expressions de V_A :

$$V_A = \frac{e + (1+jRC\omega)s}{2+jRC\omega} = (1+jRC\omega)\,s \,.$$ on obtient :

$$\frac{s}{e} = \frac{1}{1+2jRC\omega + (jRC\omega)^2} = \frac{H_0}{1+2j\lambda\frac{\omega}{\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$$

$\left\{ \begin{array}{l} H_0 = H_{max} = 1 \\ \omega_0 = \frac{1}{RC} \\ \lambda = 1 > \frac{1}{\sqrt{2}} \text{ pas de résonance.} \end{array} \right.$

Calcul de la pulsation de coupure $x_c = \frac{\omega_c}{\omega_0}$

$$|H|(x_c) = \frac{H_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \rightarrow \quad (1-x_c^2)^2 + 4x_c^2 = 2$$

$$1 - 2x_c^2 + x_c^4 + 4x_c^2 = 2 \,.$$

$$x_c^4 + 2x_c^2 - 1 = 0 \,.$$

$\Delta = 4+4 = 8$

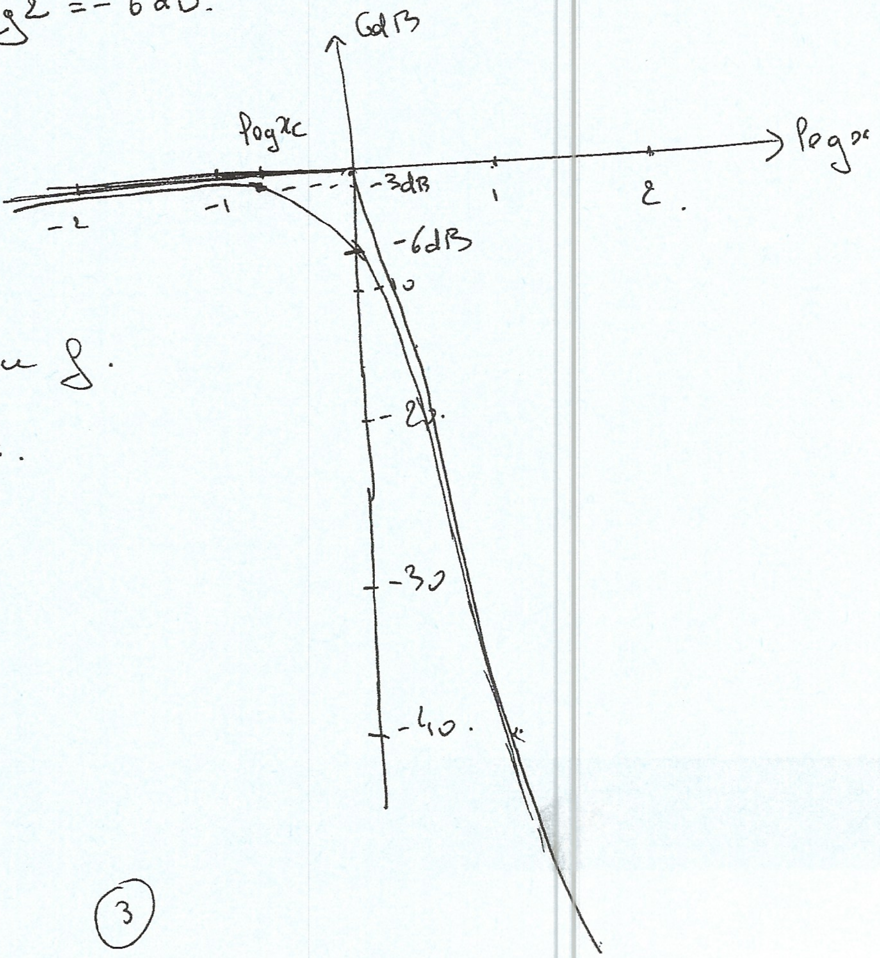
$$x_c^2 = \frac{-2+\sqrt{8}}{2} = -1+\sqrt{2} \Rightarrow x_c = \sqrt{\sqrt{2}-1} \Rightarrow \boxed{\omega_c = \sqrt{\sqrt{2}-1}\cdot\omega_0} \simeq 0{,}6\,\omega_0 \,.$$

$G_{dB}(x_c) = -3\,dB$

et $G_{dB}(x=1) = 20\log\frac{1}{2} = -20\log 2 = -6\,dB$.

$Y_{BF} = 20\log H_0 = 0$.

$Y_{HF} = 20\log H_0 - 40\log x$



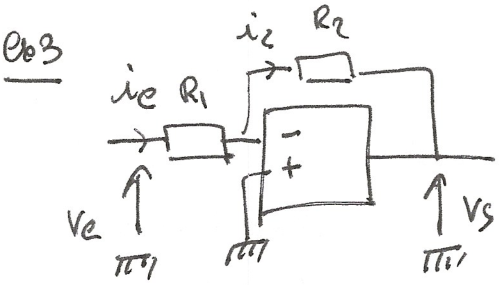
$e(t)$ = signal créneau de fréquence f.

$f < f_c \rightarrow$ pas de modification.

$f > f_c \rightarrow$ harmonique 1 "passé"

harmonique 3 atténué

Ex3



1) l'ALI est idéal $\Rightarrow i^+ = i^- = 0$.

$$V^{\ominus} = \frac{\frac{V_e}{R_1} + \frac{V_s}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = 0 \rightarrow \boxed{V_s = -\frac{R_2}{R_1} V_e.}$$

$V_e = R_1 i_e$ puisque $V^{\ominus} = 0 \rightarrow Z_e = \frac{V_e}{i_e} = R_1$

2) $\underline{V_s} = \underline{\mu}(v^+ - v^-)$ et $\begin{cases} v^+ = 0 \\ V^{\ominus} = \dfrac{\frac{V_e}{R_1} + \frac{V_s}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \dfrac{R_2 . V_e + R_1 . V_s}{R_1 + R_2} \end{cases}$

$$\underline{V_s} = \mu\left(0 - \left(\frac{R_2 V_e + R_1 V_s}{R_1 + R_2}\right)\right) \Rightarrow \underline{V_s}\left[\frac{1}{\mu} + \frac{R_1}{R_1 + R_2}\right] = -\frac{R_2 . V_e}{R_1 + R_2}.$$

$$\underline{V_s}\left[\frac{R_1 + R_2}{\underline{\mu}} + R_1\right] = -R_2 . V_e$$

$$\underline{V_s}\left[\frac{(R_1+R_2)(1+j\frac{\omega}{\omega_c})}{\mu_0} + R_1\right] = -R_2 . V_e \Rightarrow \underline{V_s}\left[R_1 + R_2 + j(R_1+R_2)\frac{\omega}{\omega_c} + R_1\mu_0\right] = -R_2 . \mu_0 . \underline{V_e}.$$

$$\Rightarrow \frac{\underline{V_s}}{\underline{V_e}} = \frac{-R_2 . \mu_0 .}{R_1 + R_2 + R_1\mu_0 + j(R_1 + R_2) . \frac{\omega}{\omega_c}}.$$

$$= \frac{-\frac{R_2 \mu_0}{R_1 + R_2 + R_1\mu_0}}{1 + j\left(\frac{R_1 + R_2}{R_1 + R_2 + R_1\mu_0} . \frac{\omega}{\omega_c}\right)} = \frac{H'_0}{1 + j\frac{\omega}{\omega'_c}}$$

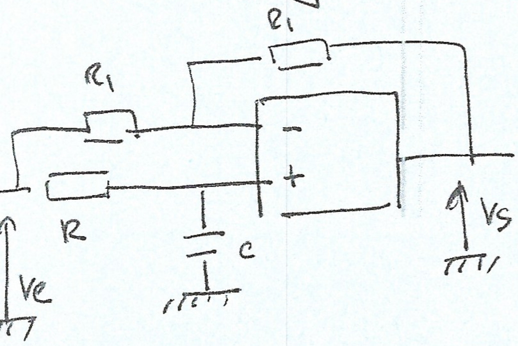
$$\boxed{H'_0 = -\frac{R_2}{R_1\left[1 + \frac{R_1 + R_2}{R_1\mu_0}\right]}}$$

$$\omega'_c = \frac{(R_1\mu_0 + R_1 + R_2)\,\omega_c}{R_1 + R_2} = \boxed{\left(1 + \frac{R_1\mu_0}{R_1 + R_2}\right)\omega_c = \omega'_c}$$

$\frac{R_1 + R_2}{R_1} \ll \mu_0 \rightarrow H'_0 \simeq -\frac{R_2}{R_1}$

$\omega'_c \rightarrow \frac{R_1\mu_0}{R_1 + R_2} . \omega_c \gg \omega_c$.

Ao4 Montage de phaseur



$$V^+ = \frac{\frac{V_e}{R} + jC\omega \cdot 0}{\frac{1}{R} + jC\omega} = \frac{V_e}{1+jRC\omega}$$

$$V^\ominus = \frac{\frac{V_e}{R_1} + \frac{V_s}{R_1}}{\frac{2}{R_1}} = \frac{V_e + V_s}{2}$$

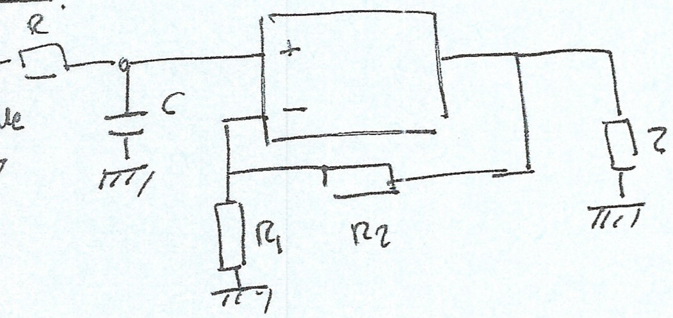
$$V^+ = V^- \Rightarrow \frac{V_e}{1+jRC\omega} = \frac{V_e + V_s}{2} \Rightarrow 2V_e = (1+jRC\omega)(V_e + V_s)$$

$$V_e(1 - jRC\omega) = (1+jRC\omega)V_s \Rightarrow \boxed{\underline{H} = \frac{V_s}{V_e} = \frac{1-jRC\omega}{1+jRC\omega}}$$

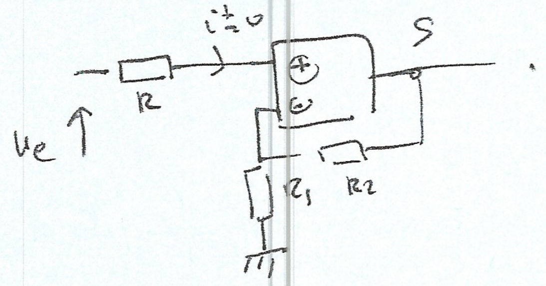
$= |\underline{H}| e^{j\varphi}$

$|\underline{H}| = 1$ et $\boxed{\varphi = -2\arctg RC\omega}$ Montage de phaseur

Ao5



BF : $-||- \Rightarrow$ —/—



$$BF : \frac{u_s}{u_e} = \frac{R_1 + R_2}{R_1}$$

$\Leftarrow$ $V^+ = u_e$ (car $i^+ = 0$).

$$V^\ominus = \frac{\frac{0}{R_1} + \frac{u_s}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1}{R_1 + R_2} u_s$$

— $-||- \Rightarrow$ —— $\Rightarrow V^+ = 0$ donc $V^\ominus \Rightarrow u_s = 0$.

Filtre passe bas

calcul de $\underline{H}$:

$$V^+ = \frac{jC\omega \cdot 0 + \frac{u_e}{R}}{\frac{1}{R} + jC\omega} = \frac{u_e}{1+jRC\omega}$$

$$V^+ = V^- \Rightarrow \frac{u_e}{1+jRC\omega} = \frac{R_1}{R_1+R_2} u_s \Rightarrow \frac{u_s}{u_e} = \frac{\frac{R_1+R_2}{R_1}}{1+jRC\omega} = \frac{H_0}{1+j\frac{\omega}{\omega_c}}$$

$H_0 = \frac{R_1 + R_2}{R_1}$ $\boxed{\omega_c = \frac{1}{RC}}$

AN $R = 100\,\Omega$

$C = 10\,\mu F = 10^{-5} F$.

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\cdot\frac{1}{RC} = \frac{1}{2\pi}\cdot\frac{1}{10^{3}} = 159\ Hz$$

$$H_0 = \frac{R_1+R_2}{R_1} = \frac{10}{1} = 10.$$

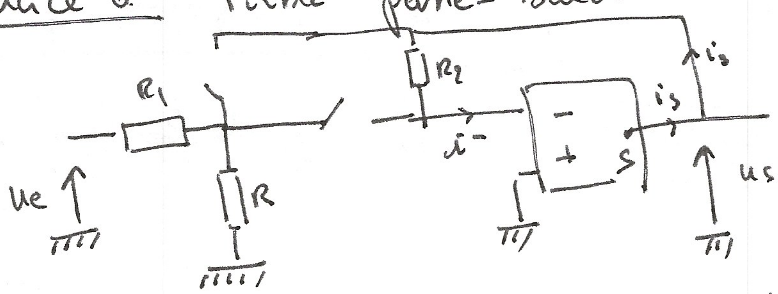
$$G_{dB} \xrightarrow{BF} 20\log H_0 = 20\,dB = Y_{BF}$$

$$\xrightarrow{HF} 20\log H_0 - 20\log x.$$

φ varie de 0 à $-\frac{\pi}{2}$.

Exercice 6. Filtre passe-bande.

BF



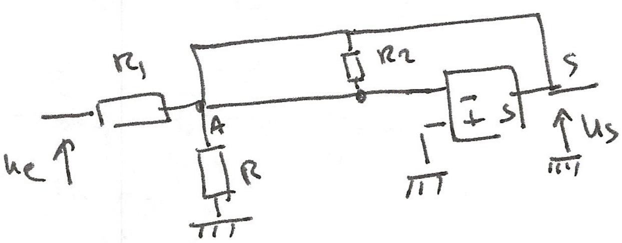
$V^+ = 0 \Rightarrow V^- = 0.$

$V^- - u_s = -R_2 i_s = -R_2 \underset{=0}{i^-} = 0$

$\Rightarrow u_s = V^- = 0.$

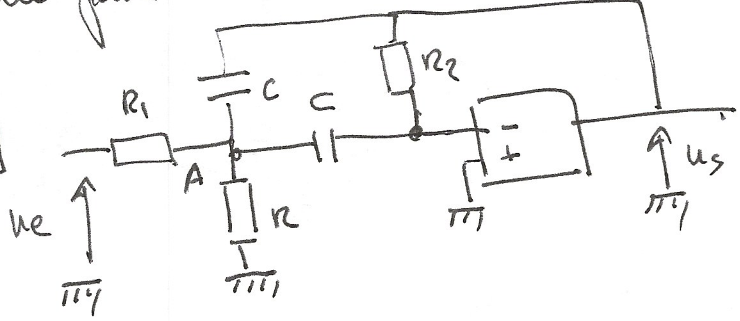
$\left.\begin{array}{l} V_A = V_S = V^- \\ \text{et } V^- = V^+ = 0 \end{array}\right\} \to V_S = 0$

HF



Filtre passe bande.

2]



$V^+ = 0$

$$V^- = \frac{\frac{u_s}{R_2} + jC\omega V_A}{\frac{1}{R_2} + jC\omega} = 0.$$

$\Rightarrow V^-$

$$V_A = -\frac{u_s}{jR_2C\omega}.$$

$$V_A = \frac{\frac{u_e}{R_1} + \frac{0}{R} + jC\omega u_s}{\frac{1}{R_1} + \frac{1}{R} + jC\omega} = \frac{u_e + jR_1C\omega u_s}{1 + \frac{R_1}{R} + jR_1C\omega} = \frac{Ru_e + jR_1RC\omega u_s}{R+R_1+jRR_1C\omega} = \frac{-u_s}{jR_2C\omega}$$

$$\left[Ru_e + jR_1RC\omega u_s\right] jR_2C\omega = -\left[R+R_1+jRR_1C\omega\right]u_s$$

$$\frac{u_s}{u_e} = \frac{-jRR_2C\omega}{R+R_1+jRR_1C\omega + RR_1R_2C^2(j\omega)^2} = \frac{-j\frac{RR_2C\omega}{R_1+R}}{1+j\frac{RR_1}{R+R_1}C\omega + \frac{RR_1R_2}{R+R_1}C^2\omega^2}$$

A identifier à. $\underline{H} = \dfrac{H_0 \cdot j\frac{\omega}{Q\omega_0}}{1 + j\frac{\omega}{Q\omega_0} + \left(j\frac{\omega}{\omega_0}\right)^2}$

d'où $\begin{cases} \dfrac{1}{\omega_0^2} = \dfrac{R R_1 R_2}{R+R_1} \cdot C^2 \\ \dfrac{1}{Q\omega_0} = \dfrac{R R_1 C}{R+R_1} \\ \dfrac{H_0}{Q\omega_0} = \dfrac{-R R_2 C}{R+R_1} \end{cases}$ $\rightarrow$ $\boxed{\omega_0 = \sqrt{\dfrac{R+R_1}{R R_1 R_2}} \cdot \dfrac{1}{C}.}$

$\rightarrow$ $\boxed{H_0 = -\dfrac{R_2}{R_1}}$

et $\boxed{Q = \sqrt{\dfrac{R_2(R+R_1)}{R R_1}}.}$

4] $H_0 = -2$ $Q = 10$

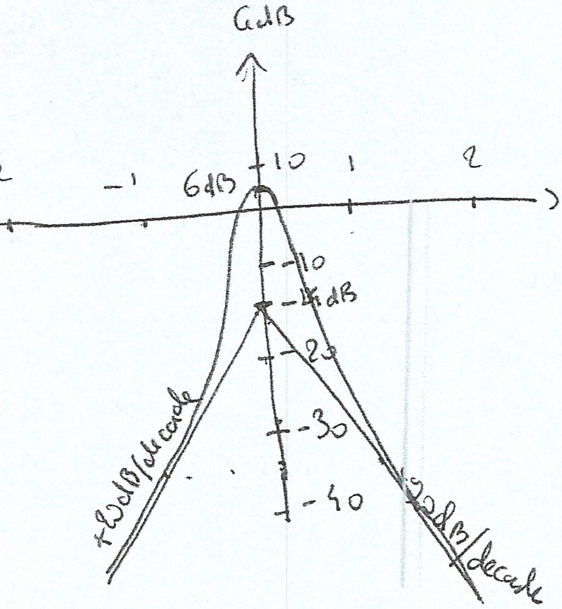
$\underline{H} \xrightarrow{BF} \dfrac{H_0}{Q} \cdot jx \Rightarrow Y_{BF} = 20 \log \dfrac{|H_0|}{Q} + 20 \log x.$ et $\varphi_{BF} = -\pi + \dfrac{\pi}{2} = -$

$\underline{H} \xrightarrow{HF} \dfrac{H_0}{Q} \cdot \dfrac{jx}{(jx)^2} = \dfrac{H_0}{Q} \cdot \dfrac{1}{jx} \rightarrow Y_{HF} = 20 \log \dfrac{|H_0|}{Q} - 20 \log x$ et $\varphi_{HF} = -$

$I \left| \begin{array}{l} x = 1. \\ 20 \log \dfrac{|H_0|}{Q}. \end{array} \right.$

AN $20 \log \dfrac{|H_0|}{Q} = 20 \log \dfrac{2}{10} = 20 \log 2 - 20 \log 10$
$= 6 - 20 = -14\,dB.$

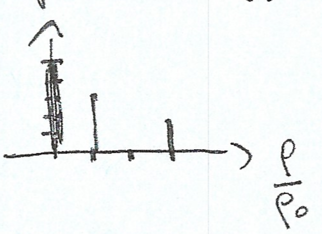
$\begin{cases} G_{max} = 20 \log |H_0| = 20 \log 2 = 6\,dB. \\ \varphi(x=1) = -\pi. \end{cases}$



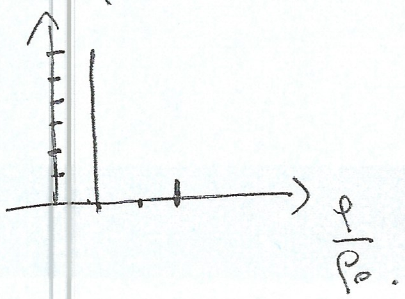
$U_e(t) = 5 + 3\cos(2\pi f_0 t) - 2\sin(2\pi(3f_0)t)$

$U_s(t) = 0 + 6\cos(2\pi f_0 t + \pi) - \underbrace{2 \cdot |\underline{H}(x=3)|}_{0,15} \sin(2\pi(3f_0)$ $+ \varphi(x$ $+\dfrac{\pi}{2}$

Spectre de Ue(t)

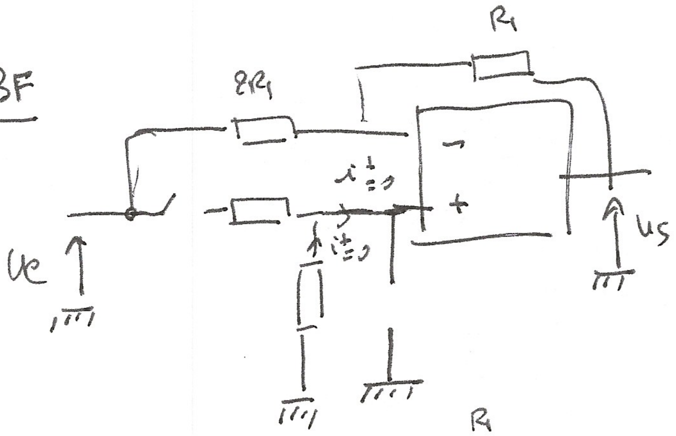


Spectre de Us(t)



Exo 7 Filtre réjecteur.

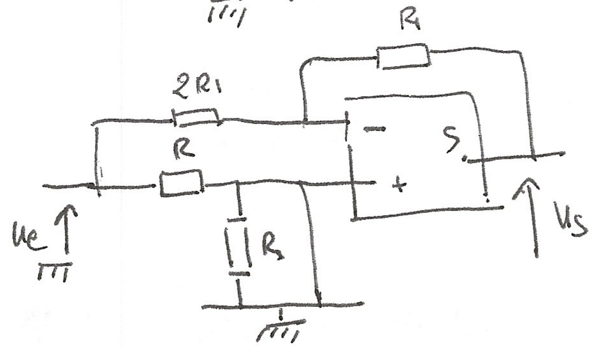
1°) BF



$v^+ = 0$ $\qquad v^{\ominus} = \dfrac{\dfrac{u_e}{2R_1} + \dfrac{u_s}{R_1}}{\dfrac{1}{2R_1} + \dfrac{1}{R_1}}$

$v^+ = v^- \Rightarrow \boxed{u_s = -\dfrac{u_e}{2}}$

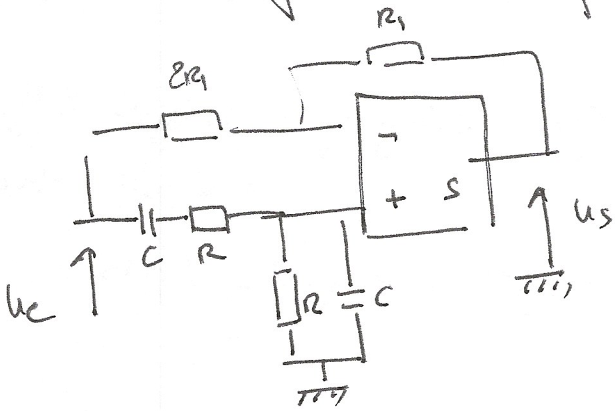
HF



$v^+ = 0$ et $v^{\ominus}$ a tjs la m̂ expression

$\boxed{u_s = -\dfrac{u_e}{2}}$

2°) Calcul de la fonction de transfert:



$$v^{\ominus} = \frac{\dfrac{u_e}{2R_1} + \dfrac{u_s}{R_1}}{\dfrac{1}{2R_1} + \dfrac{1}{R_1}} = \frac{u_e + u_s}{3}$$

$$v^{\oplus} = \frac{\dfrac{u_e}{R + \frac{1}{jC\omega}} + \dfrac{0}{Z_{eq}}}{\dfrac{1}{R + \frac{1}{jC\omega}} + \dfrac{1}{Z_{eq}}}$$

$$v^+ = \frac{\dfrac{jC\omega\, u_e}{1 + jRC\omega}}{\dfrac{jC\omega}{1+jRC\omega} + \dfrac{1}{R} + jC\omega} = \frac{jC\omega\, u_e}{jC\omega + (1 + jRC\omega)\left(\frac{1}{R} + jC\omega\right)} = \frac{jRC\omega\, u_e}{1 + 3jRC\omega + (jRC\omega)^2}$$

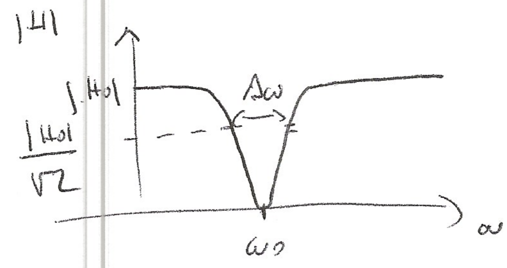
$$= \frac{u_e + 2u_s}{3} \quad \ldots$$

Après calculs

$$\frac{u_s}{u_e} = \frac{-\frac{1}{2}\left(1 + (jRC\omega)^2\right)}{1 + 3jRC\omega + (jRC\omega)^2} = \frac{H_0\,(1 - x^2)}{1 - x^2 + j\frac{x}{Q}}$$

$$\begin{cases} H_0 = -\dfrac{1}{2} \\ \dfrac{1}{\omega_0} = RC \rightarrow \omega_0 = \dfrac{1}{RC} \\ \dfrac{1}{\omega_0 Q} = 3RC \rightarrow Q = \dfrac{1}{3} \end{cases}$$

$$\underline{H} = \frac{H_0}{1 + \frac{jx/Q}{1-x^2}} = \frac{H_0}{1 + \frac{1}{Q\left(\frac{1}{x} - x\right)}} = \frac{H_0}{1 + \frac{1}{jQ\left(x - \frac{1}{x}\right)}}.$$



$x \to 0 \quad \underline{H} \to H_0. \qquad Y_{BF} = 20 \log H_0$

$x \to \infty \quad \underline{H} \to H_0 \qquad Y_{HF} = 20 \log H_0$

$x = 1 \quad \underline{H} \to 0. \qquad G_{dB} \to -\infty$

$$|\underline{H}(x_1)| = |\underline{H}(x_2)| = \frac{|H_{max}|}{\sqrt{2}} = \frac{H_0}{\sqrt{2}} \quad \to \quad 2 = 1 + \frac{1}{Q^2\left(x - \frac{1}{x}\right)^2}$$

$$\Rightarrow \left(x - \frac{1}{x}\right) = \pm \frac{1}{Q} \quad \Rightarrow \quad \frac{\omega_0}{\Delta\omega} = Q.$$