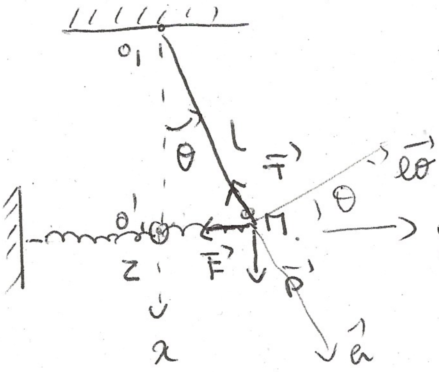
eo3 Correction TD Moment cinétique -



TMC $\dfrac{d\vec{L_{O_1}}}{dt} = \vec{O_1M}\wedge\vec{F} + \vec{O_1M}\wedge\vec{T} + \vec{O_1M}\wedge\vec{P}$

$\vec{L_{O_1}(M)} = \vec{O_1M}\wedge m\vec{v} = L\vec{e_r}\wedge m\left(L\dot{\theta}\vec{e_\theta}\right)$

$= mL^2\dot{\theta}\vec{e_z}$

$\vec{\mathcal{M}_{O_1}(P)} = \vec{O_1M} \wedge \vec{P} = L\vec{e_r}\wedge\left(mg\cos\theta\,\vec{e_r} - mg\sin\theta\,\vec{e_\theta}\right)$

$= -mgL\sin\theta\,\vec{e_z}$

$\vec{F} = -k\left(l_0 + y - l_0\right)\vec{e_y} = -kL\sin\theta\,\vec{e_y}$.

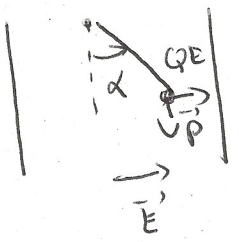
$\vec{O_1M}\wedge\vec{F} = L\left(\cos\theta\,\vec{e_x} + \sin\theta\,\vec{e_y}\right)\wedge\left(-kL\sin\theta\,\vec{e_y}\right) = -kL^2\sin\theta\cos\theta\,\vec{e_z}$

TMC $mL^2\ddot{\theta} = -mgL\sin\theta - kL^2\sin\theta\cos\theta$ avec $\sin\theta \simeq \theta$, $\cos\theta \simeq 1$.

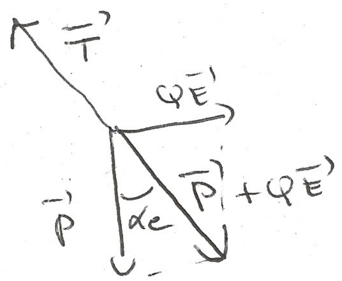
$$\boxed{\ddot{\theta} + \left(\frac{g}{L} + \frac{k}{m}\right)\theta = 0}$$

Oscillateur harmonique et $\boxed{\omega_0 = \sqrt{\dfrac{k}{m} + \dfrac{g}{L}}}$

eo4. Pendule électrostatique.

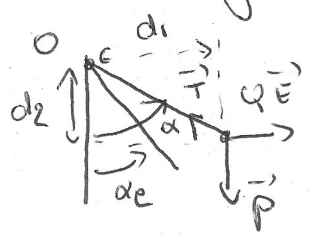


A l'équilibre $\vec{P} + Q\vec{E} + \vec{T} = \vec{0}$.



$$\boxed{\tan\alpha = \frac{QE}{mg}.}$$

Equation des petites oscillations :



$\dfrac{d\vec{L_O}}{dt} = mL^2\ddot{\alpha}\,\vec{e_z}$

$\vec{\mathcal{M}_O(\vec{P})} = -mgd_1\vec{e_z} = -mgL\sin\alpha\,\vec{e_z}$

$\vec{\mathcal{M}_O}(Q\vec{E}) = +QE\cdot d_2\,\vec{e_z} = QEL\cos\alpha\,\vec{e_z}$

$\vec{\mathcal{M}_O}(\vec{T}) = \vec{0}$

$$mL^2\ddot{\alpha} = -mgL\sin\alpha + qEL\cos\alpha.$$

$$\ddot{\alpha} + \frac{g}{L}\sin\alpha - \frac{qE}{mL}\cos\alpha = 0 \qquad (1)$$

avec
$$\left\{\begin{array}{l} \dfrac{qE}{mL} = \dfrac{qE}{mg}\cdot\dfrac{g}{L} = tg\,\alpha_E\cdot\dfrac{g}{L} \\[2ex] \sin\alpha = \sin\alpha_E + (\alpha-\alpha_E)\cos\alpha_E \\ \cos\alpha = \cos\alpha_E - (\alpha-\alpha_E)\sin\alpha_E \end{array}\right.$$

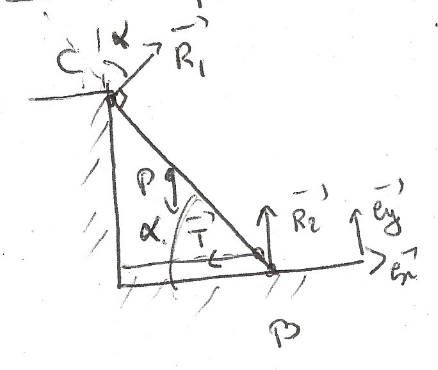
d'où (1) devient :
$$\ddot{\alpha} + \frac{g}{L}\Big[\sin\alpha_E + (\alpha-\alpha_E)\cos\alpha_E - tg\,\alpha_E\big(\cos\alpha_E - (\alpha-\alpha_E)\sin\alpha_E\big)\Big] = 0$$

$$\ddot{\alpha} + \frac{g}{L}\Big[\underbrace{\sin\alpha_E - tg\,\alpha_E\cdot\cos\alpha_E}_{=0}\Big] + \frac{g}{L}\Big[\underbrace{\cos\alpha_E + tg\,\alpha_E\cdot\sin\alpha_E}_{\underbrace{\left(\cos\alpha_E + \frac{\sin^2\alpha_E}{\cos\alpha_E}\right)}_{\frac{1}{\cos\alpha_E}}}\Big](\alpha-\alpha_E) = 0$$

d'où $\ddot{\alpha} + \dfrac{g}{L\cos\alpha_E}(\alpha-\alpha_E) = 0 \quad \rightarrow \quad \boxed{\omega_0^2 = \dfrac{g}{L\cos\alpha_E}}$

$$\frac{1}{\cos\alpha_E} = \sqrt{1+tg^2\alpha_E} = \sqrt{1+\frac{q^2E^2}{m^2g^2}}.$$

Ex 5. Equilibre d'une échelle.



$\vec P$ s'applique au centre d'inertie de l'échelle.
$\vec{R_2}$ réaction du sol sur l'échelle en B (pas de frottement) est ⊥ sol.
$\vec{R_1}$ = réaction du mur / l'échelle $= -\vec{R_1'}$ où $\vec{R_1'}$ est la réaction de l'échelle sur le mur donc ⊥ échelle. ⇒ d'où la direction de $\vec{R_1}$.

A l'équilibre $\sum \vec{F}/\text{échelle} = 0 \qquad \sum \vec{\mathcal{M}}(\vec F) = \vec{0}$.

$\vec{R_2} = R_2 \vec{e_y}$ 　　　3 inconnues 　R_2, R_1, T.

$\vec{R_1} = R_1 \sin\alpha \vec{e_x} + R_1 \cos\alpha \vec{e_y}$

$\vec{T} = -T \vec{e_x}$

$\vec{P} = -mg \vec{e_y}$

$\sum \vec{F}/\text{echelle} = \vec{0} \rightarrow$ proj$/\vec{e_x}$: $R_1 \sin\alpha - T = 0$ (1)

proj$/\vec{e_y}$ 　$R_2 - mg + R_1 \cos\alpha = 0$. (2)

$\sum \vec{\mathcal{M}}_B(\vec{F}) = 0 \rightarrow \vec{\mathcal{M}}_B(\vec{P}) + \vec{\mathcal{M}}_B(\vec{R_1}) = \vec{0}$ (3)

$\vec{\mathcal{M}}_B(\vec{P}) = \vec{BG} \wedge \vec{P} = \frac{L}{2}(-\cos\alpha \vec{e_x} + \sin\alpha \vec{e_y}) \wedge -mg \vec{e_y}$.

$= \frac{L}{2} \cos\alpha \, g \cdot \vec{e_z}$.

$\vec{\mathcal{M}}_B(\vec{R_1}) = \vec{BC} \wedge \vec{R_1} = \frac{L}{1}(-\cos\alpha \vec{e_x} + \sin\alpha \vec{e_y}) \wedge (R_1 \sin\alpha \vec{e_x} + R_1 \cos\alpha \vec{e_y})$

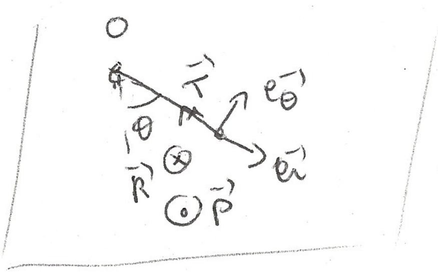
$= L \begin{vmatrix} -\cos\alpha \\ \sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} R_1 \sin\alpha \\ R_1 \cos\alpha \\ 0 \end{vmatrix} = L(-R_1 \cos^2\alpha - R_1 \sin^2\alpha)_{\vec{e_z}} = -R_1 \cdot L \vec{e_z}$.

d'où 　$-R_1 \cdot L + \frac{L}{2} \cos\alpha \cdot mg = 0 \rightarrow \boxed{R_1 = \frac{mg \cos\alpha}{2}}$

(1) $\rightarrow T = R_1 \sin\alpha = \boxed{\frac{mg \cos\alpha \sin\alpha}{2} = T}$

(2) $\boxed{R_2 = mg - R_1 \cos\alpha = mg - \frac{mg}{2} \cdot \cos^2\alpha.}$

Q6



$\vec{P}$ et $\vec{R}$ ⊥ support 　$\rho(t) = \rho_0 - v_0 t$.

$\vec{L_O}(n) = m \rho^2 \dot{\theta} \vec{e_z}$ 　et 　$\sum \vec{\mathcal{M}}_O(\vec{F}/n)$

$= \vec{On} \wedge \underbrace{(\vec{P} + \vec{R})}_{=0} + \underbrace{\vec{On} \wedge \vec{T}}_{=0 \text{ car } \vec{T} // \vec{On}} = 0 \rightarrow m\rho^2\dot{\theta} = \text{cste}$

$\omega(t) = \frac{\rho_0^2 \omega_0}{(\rho - v_0 t)^2}$ -