

Correction DS 2.

Serie générale.

Exercice 1

$$1) A = \sum_{k=0}^n 5k + 2 + \left(\frac{3}{4}\right)^k = 5 \times \frac{n(n+1)}{2} + 2(n+1) + \frac{1 - \left(\frac{3}{4}\right)^{n+1}}{1 - \frac{3}{4}}$$

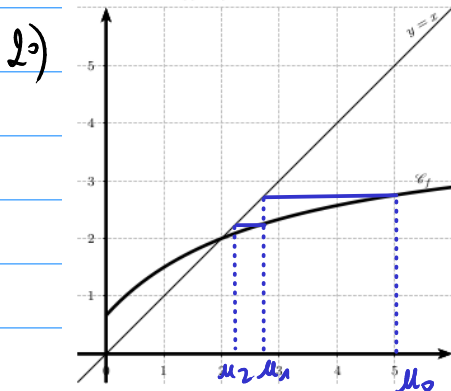
$$A = \frac{(n+1)(5n+4)}{2}$$

$$2) B = \sum_{k=n}^{2n} k = \frac{2n(2n+1)}{2} - \frac{n(n-1)}{2} = \frac{n(4n+2-n+1)}{2} = \frac{3n(n+1)}{2}$$

Exercice 2.

(u_n) définie par $u_0 = 5$ et $\forall n \in \mathbb{N}, u_{n+1} = \frac{4u_n + 2}{u_n + 3}$

$$1) u_1 = \frac{4 \times 5 + 2}{5 + 3} = \frac{22}{8} = \frac{11}{4} = 2,75.$$



$$3) \forall n \in \mathbb{N}, v_n = \frac{u_n - 2}{u_n + 1}$$

a) Soit $n \in \mathbb{N}$

$$v_{n+1} = \frac{u_{n+1} - 2}{u_{n+1} + 1} = \frac{\left(\frac{4u_n + 2}{u_n + 3} - 2\right) \times (u_n + 3)}{\left(\frac{4u_n + 2}{u_n + 3} + 1\right) \times (u_n + 3)}$$

$$v_{n+1} = \frac{4u_n + 2 - 2u_n - 6}{4u_n + 2 + u_n + 3} = \frac{2u_n - 4}{5u_n + 5}$$

$$\text{donc } v_{n+1} = \frac{2}{5} \times \frac{u_n - 2}{u_n + 1} = \frac{2}{5} v_n$$

Ainsi (v_n) est géométrique de raison $\frac{2}{5}$

$$3) b) \text{ et } v_0 = \frac{5-2}{5+1} = \frac{1}{2}.$$

$$\text{Donc } \forall n \in \mathbb{N}, v_n = \frac{1}{2} \times \left(\frac{2}{5}\right)^n$$

$$3) c) \forall n \in \mathbb{N}, v_n = \frac{u_n - 2}{u_n + 1} \Leftrightarrow v_n(u_n + 1) = u_n - 2$$

$$\Leftrightarrow u_n(v_n - 1) = -v_n - 2$$

$$\Leftrightarrow u_n = \frac{v_n + 2}{1 - v_n}$$

3)d) Ainsi, $\forall n \in \mathbb{N}, u_n = \frac{\frac{1}{2} \times \left(\frac{2}{5}\right)^n + 2}{1 - \frac{1}{2} \times \left(\frac{2}{5}\right)^n}$

Exercice 3

$u_0 = 0, u_1 = 1$ et, $\forall n \in \mathbb{N}, u_{n+2} = 5u_{n+1} - 4u_n$

1) $u_2 = 5 \times u_1 - 4u_0 = 5$

$u_3 = 5 \times u_2 - 4u_1 = 25 - 4 = 21$

2) Première approche

a) $\forall n \in \mathbb{N}, v_n = u_{n+1} - u_n$

Soit $n \in \mathbb{N}, v_{n+1} = u_{n+2} - u_{n+1} = 4u_{n+1} - 4u_n = 4v_n$

Puis (v_n) est géométrique de raison 4, $v_0 = u_1 - u_0 = 1$

donc $\forall n \in \mathbb{N}, v_n = 4^n$

b) $\forall n \in \mathbb{N}, w_n = u_{n+1} - 4u_n$

Soit $n \in \mathbb{N}, w_{n+1} = u_{n+2} - 4u_{n+1} = u_{n+1} - 4u_n = w_n$

Puis (w_n) est constante donc $\forall n \in \mathbb{N}, w_n = w_0 = u_1 - 4u_0$

$\forall n \in \mathbb{N}, w_n = 1$

c) On a, $\forall n \in \mathbb{N}, v_n = u_{n+1} - u_n$

$w_n = u_{n+1} - 4u_n$

donc $v_n - w_n = 3u_n$

donc

$u_n = \frac{v_n - w_n}{3} = \frac{4^n - 1}{3}$

3) Seconde approche

a) Soit $n \in \mathbb{N}$

$u_{n+2} = 4u_n + 1 \Leftrightarrow u_{n+2} = 4u_{n+1} + 1$

$\Leftrightarrow u_{n+2} = 4u_{n+1} + (u_{n+1} - 4u_n)$

$\Leftrightarrow u_{n+2} = 5u_{n+1} - 4u_n$

$$3) b) \quad x = 4x + 1 \Leftrightarrow x = -\frac{1}{3}.$$

$$\forall n \in \mathbb{N}$$

$$\begin{cases} u_{n+1} = 4u_n + 1 \\ -\frac{1}{3} = 4 \times \left(-\frac{1}{3}\right) + 1 \end{cases}$$

donc $u_{n+1} + \frac{1}{3} = 4\left(u_n + \frac{1}{3}\right)$ donc $\left(u_n + \frac{1}{3}\right)$ est géométrique de raison 4
 $u_0 + \frac{1}{3} = \frac{1}{3}.$

$$\text{Ainsi, } \forall n \in \mathbb{N}, \quad u_n + \frac{1}{3} = \frac{1}{3} \times 4^n.$$

$$\Leftrightarrow u_n = \frac{4^n - 1}{3}$$